# Truth Tables

Networks and Embedded Systems

First Grade Level

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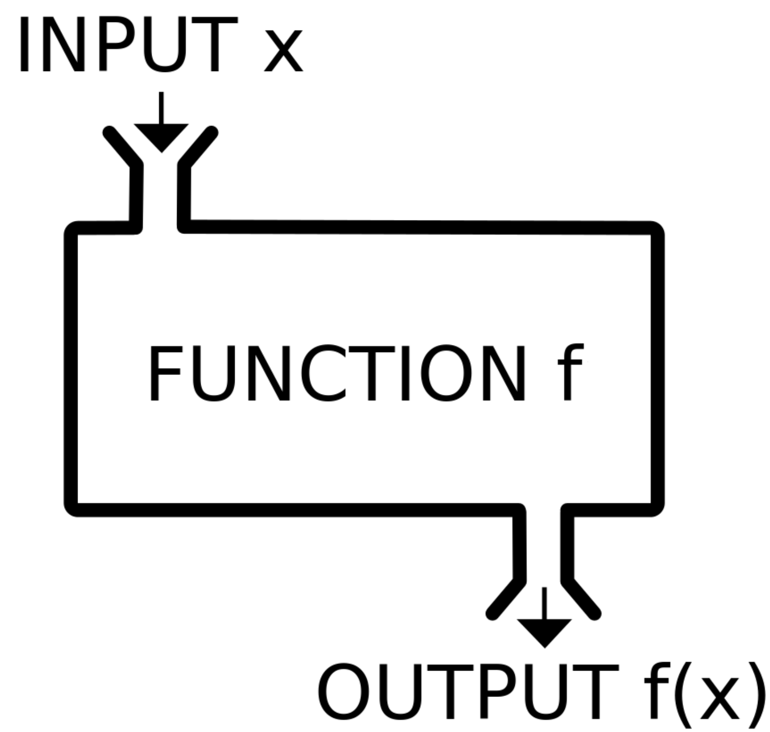
# Truth Tables (1)

- Functions
  - Process input
  - Produce output
  - They have
    - Parameters
      - Input
  - They return
    - Results
      - Output



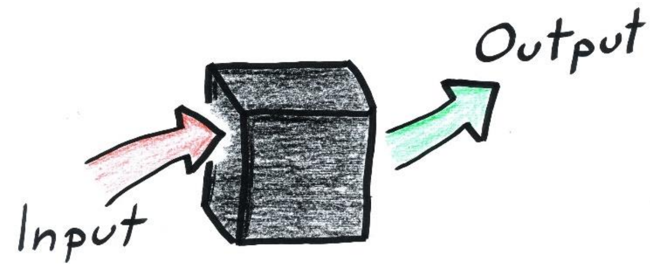
# Truth Tables (2)

- Logical Functions
  - Are expression of
    - Parameters (a, b, c, …)
    - Operators (¬, ∧, ∨, …)
    - Functions ($\varphi$, $\psi$, $\chi$, …)
  - Alternative terms
    - Logic function
    - Boolean function
    - Switching function

$$\underbrace{\varphi(a,b)}_{\text{Parameters}} = \underbrace{a \wedge b}_{\text{Expression}}$$

Parameters

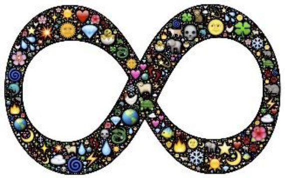
Expression




| 0, 1 | $\varphi$(a,b) | 0 |
|---|---|---|
| Arguments | Function | Result |

# Truth Tables (3)

- Logical Functions (continued)
  - Arithmetic and truth functions are different
    - Arithmetic functions are infinite
    - Truth functions are finite



$f(1,1) = 2$
$f(1,2) = 3$
$f(1,3) = 4$
$f(1,4) = 5$
…
Arguments never end

Arithmetic function $f(x,y) = x+y$

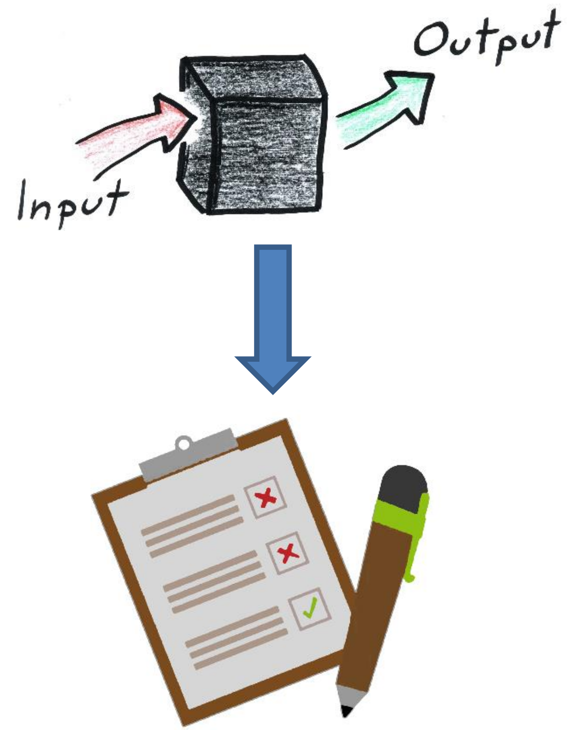
$\varphi(0,0) = 0$
$\varphi(0,1) = 0$
$\varphi(1,0) = 0$
$\varphi(1,1) = 1$

No arguments any more

Truth function $\varphi(a,b) = a \wedge b$

# Truth Tables (4)

- Logical Functions (continued)
  - A truth table can be made
    - All possible inputs (arguments)
    - All possible outputs (results)
  - Alternative terms
    - Truth table
    - Switching table
    - State table



# Truth Tables (5)

- Logical Functions (finished)
  - Example
    - $\varphi(a,b) = a \wedge b$
  - Description
    - Two parameters
      - a, b
    - Four arguments
      - (0,0), (0,1), (1,0), (1,1)
    - One result
      - 0 or 1

| a | b | φ(a,b) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Truth Tables (6)

- Examples
  - Functions with one parameter: φ(a)
  - Functions with two parameters: φ(a,b)

| a | φ(a) |
|---|---|
| 0 | … |
| 1 | … |

| a | b | φ(a,b) |
|---|---|---|
| 0 | 0 | … |
| 0 | 1 | … |
| 1 | 0 | … |
| 1 | 1 | … |

# Truth Tables (7)

- Construction
  - Number of columns
    - Number of parameters: $n$
    - Plus number of results
  - Number of rows
    - Number of arguments: $2^n$
  - First column
    - Fifty-fifty: ½ column 0, ½ column 1

# Truth Tables (8)

- Construction (continued)
  - Second column
    - Fifty-fifty but twice as fast: ¼ *0*, ¼ *1*, ¼ *0*, ¼ *1*
  - And so on …
  - Checklist
    - First rows starts with: 0 0 0 0 …
    - Last rows ends with: 1 1 1 1 …
    - Last column is alternating*:* 0 1 0 1 0 1 …
    - Sequence of rows: binary natural numbers (0 1 2 3 …)

# Truth Tables (9)

- Functions with four parameters: φ(a,b,c,d)

| a | b | c | d |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |

continued on the right

| a | b | c | d |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Truth Tables (10)

- Logical Equivalence
  - Truth tables are identical
  - $\varphi \leftrightarrow \psi$

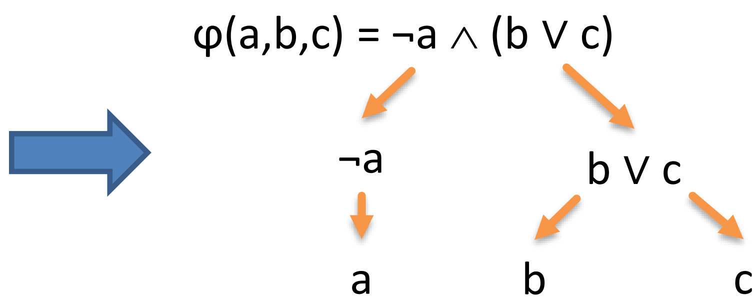
| a | b | φ(a,b) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| a | b | ψ(a,b) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# Truth Tables (11)

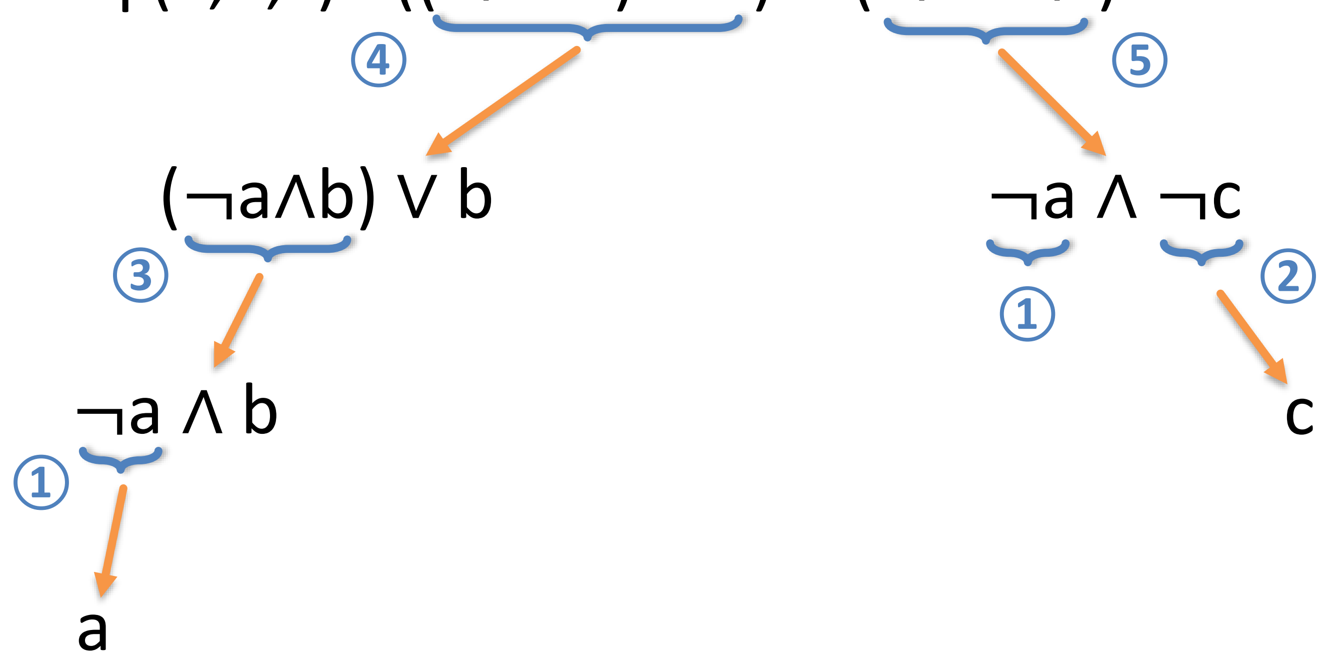
- Creation (first method)
  - Mathematician's method
  - Term by term
    - Terms: compose the expression of a function
    - Functions: span a tree of terms

$\varphi(a,b,c) = \underbrace{\underbrace{\neg a}_{1^{st}\ \text{term}} \land \underbrace{(b \lor c)}_{2^{nd}\ \text{term}}}_{3^{rd}\ \text{term}}$



# Truth Tables (12)

- Creation (first method, continued)
  - Example: φ(a,b,c) =((¬a∧b) ∨ b) ∨ (¬a∧¬c)



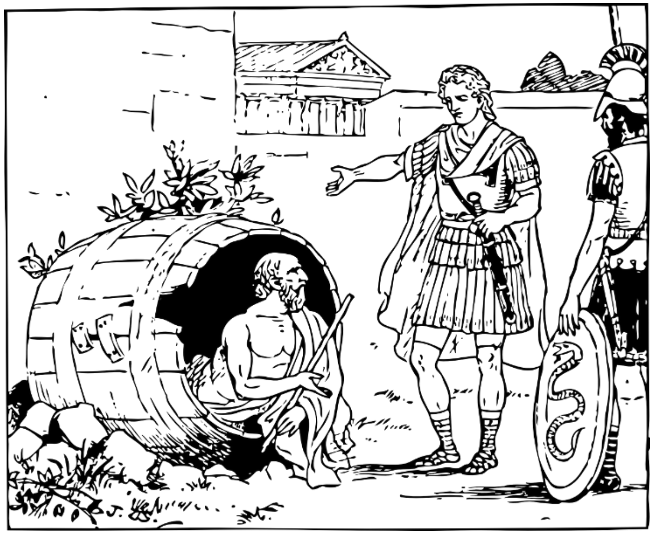
# Truth Tables (13)

- Creation (first method, finished)
  - Example: $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

| | | | ① | ② | ③ | ④ | ⑤ | φ(a,b,c) |
|---|---|---|---|---|---|---|---|---|
| a | b | c | ¬a | ¬c | ①∧b | ③∨b | ①∧② | ④∨⑤ |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |

# Truth Tables (14)

- Creation (second method)
  - Philosopher's method
    - Put each symbol in a column
    - Fill in the arguments
    - Perform the operations
      - Fill the column with the results
      - The last column shows the result of the truth function.



| a | b | a | ∧ | b |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

# Truth Tables (15)

- Creation (second method, continued)
  - Example: φ(a,b,c) = ((¬a∧b) ∨ b) ∨ (¬a∧¬c)

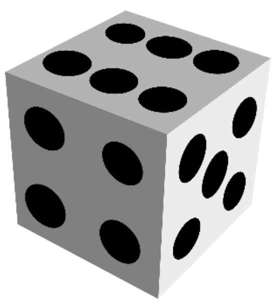
| Sequence | | | | | 2 | 1 | 4 | 3 | | 6 | 5 | | 12 | | 8 | 7 | 11 | 10 | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | b | c | ( | ( | ¬ | a | ∧ | b | ) | ∨ | b | ) | ∨ | ( | ¬ | a | ∧ | ¬ | c | ) |
| 0 | 0 | 0 | | | 1 | 0 | 0 | 0 | | 0 | 0 | | 1 | | 1 | 0 | 1 | 1 | 0 | |
| 0 | 0 | 1 | | | 1 | 0 | 0 | 0 | | 0 | 0 | | 0 | | 1 | 0 | 0 | 0 | 1 | |
| 0 | 1 | 0 | | | 1 | 0 | 1 | 1 | | 1 | 1 | | 1 | | 1 | 0 | 1 | 1 | 0 | |
| 0 | 1 | 1 | | | 1 | 0 | 1 | 1 | | 1 | 1 | | 1 | | 1 | 0 | 0 | 0 | 1 | |
| 1 | 0 | 0 | | | 0 | 1 | 0 | 0 | | 0 | 0 | | 0 | | 0 | 1 | 0 | 1 | 0 | |
| 1 | 0 | 1 | | | 0 | 1 | 0 | 0 | | 0 | 0 | | 0 | | 0 | 1 | 0 | 0 | 1 | |
| 1 | 1 | 0 | | | 0 | 1 | 0 | 1 | | 1 | 1 | | 1 | | 0 | 1 | 0 | 1 | 0 | |
| 1 | 1 | 1 | | | 0 | 1 | 0 | 1 | | 1 | 1 | | 1 | | 0 | 1 | 0 | 0 | 1 | |

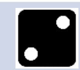
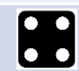
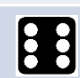
# Truth Tables (16)

- Don’t-Care Terms
  - Truth tables can be incomplete
  - Length of truth table is fixed: $2^n$
  - Unused rows are called don’t-care terms
  - They must not be omitted
  - They are marked by X

# Truth Tables (17)

- Don’t-Care Terms (continued)
  - Example: What face of a dice has six pips?



| n | a | b | c | φ(a,b,c) | Face |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X | invalid |
| 1 | 0 | 0 | 1 | 0 |  |
| 2 | 0 | 1 | 0 | 0 |  |
| 3 | 0 | 1 | 1 | 0 |  |
| 4 | 1 | 0 | 0 | 0 |  |
| 5 | 1 | 0 | 1 | 0 |  |
| 6 | 1 | 1 | 0 | 1 |  |
| 7 | 1 | 1 | 1 | X | invalid |

# Truth Tables (18)

- Conclusion
  - Truth tables describe logic functions
    - One function has exactly one truth table
  - Truth tables are not unique
    - Many functions have the same truth table
    - These functions are logically equivalent
  - Some of these functions can easily be found
    - Truth table → logic function -> digital circuit