

# Truth Tables

Networks and Embedded Software

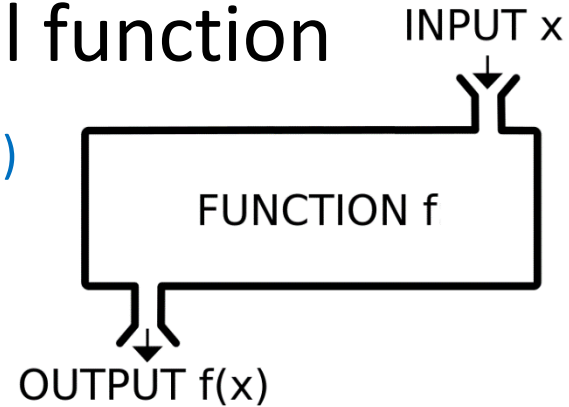
Module 3.2.2

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# Truth Tables (1)

- Representation of a logical function

- All possible inputs (arguments)
- All possible outputs (results)



- Example

–  $\varphi(a,b) = a \wedge b$  

- Two parameters
  - a, b
- Four arguments
  - (0,0), (0,1), (1,0), (1,1)

a	b	$\varphi(a,b)$
0	0	0
0	1	0
1	0	0
1	1	1

# Truth Tables (2)

- Functions with one parameter:  $\varphi(a)$

a
0
1

- Functions with two parameters:  $\varphi(a,b)$

a	b
0	0
0	1
1	0
1	1

# Truth Tables (3)

- Construction guidance
  - Number of columns: number of parameters  $n$
  - Number of rows:  $2^n$
  - Start value:  $0$
  - First column: fifty-fifty ( $\frac{1}{2}$  column  $0$  then  $\frac{1}{2}$  column  $1$ )
  - Second column: twice as fast ( $\frac{1}{4}$   $0$ ,  $\frac{1}{4}$   $1$ ,  $\frac{1}{4}$   $0$ ,  $\frac{1}{4}$   $1$ )
  - And so on ...
  - Last column:  $0$  and  $1$  alternate ( $010101\dots$ )

# Truth Tables (4)

- Functions with four parameters:  $\varphi(a,b,c,d)$

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

a	b	c	d
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

continues on the next table

# Logical Equivalence

- Truth functions are logically equivalent if their truth tables are identical.
- $\varphi \leftrightarrow \psi$  ( $\varphi$  and  $\psi$  are logically equivalent)

a	b	$\varphi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

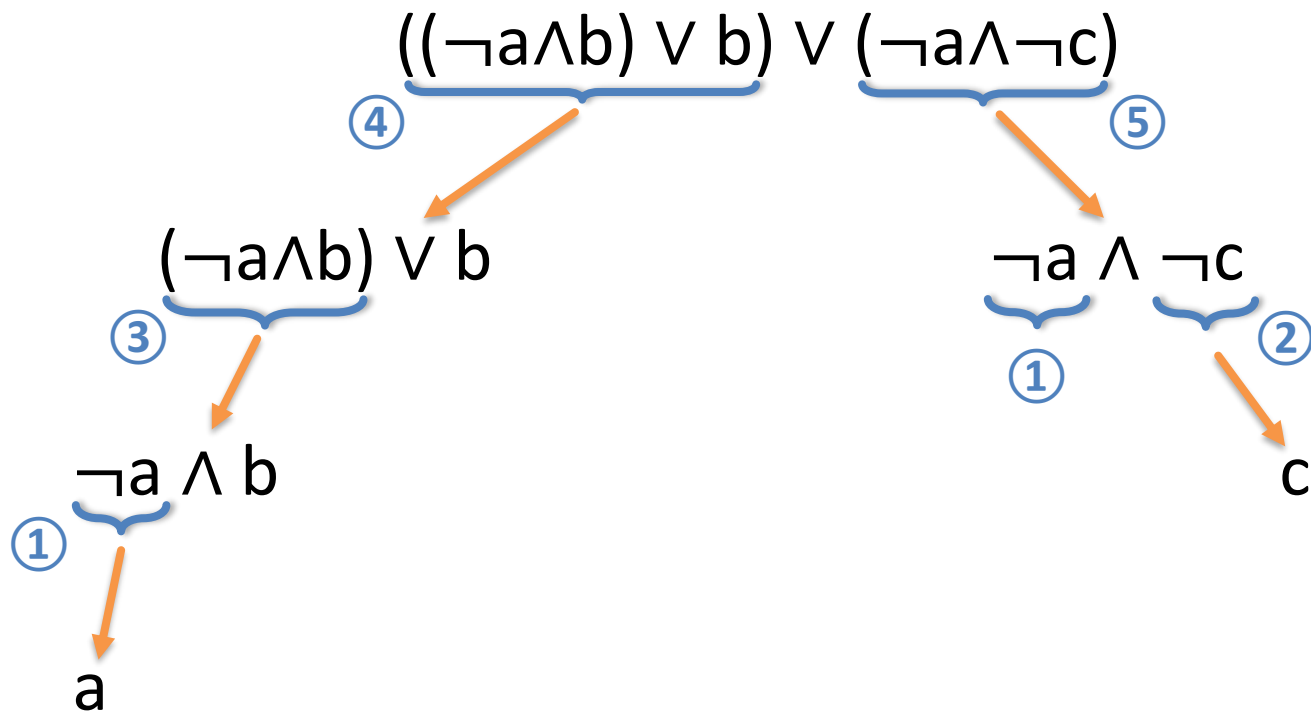
a	b	$\psi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

# Truth Functions (1)

- Also called
  - Logic functions
  - Boolean functions
- Domain
  - $\varphi : \{0,1\}^n \rightarrow \{0,1\}$
- Calculation
  - Term by term
    - Term: component of a function or expression

# Truth Functions (2)

- Functions form a tree of terms





# Truth Functions (3)

- Example:  $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

			①	②	③	④	⑤	$\varphi(a,b,c)$
a	b	c	$\neg a$	$\neg c$	① $\wedge$ b	③ $\vee$ b	① $\wedge$ ②	④ $\vee$ ⑤
0	0	0	1	1	0	0	1	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	0	1	1	0	1
1	0	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	1	0	1
1	1	1	0	0	0	1	0	1

# Truth Functions (4)

- Conclusion
  - One function has exactly one truth table
  - Are completely characterized by their truth table
  - Many functions can have the same truth table
    - These functions are logically equivalent
  - One of these truth functions can easily be found
    - Disjunctive Normal Form (DNF)

# Don't-Care Terms

- Truth tables can be incomplete.
  - Example
    - We want to use the digits 0 ... 9
    - We need four variables:  $2^4=16$
    - Six rows are unused: 10 ... 15
- Unused rows are called *don't-care terms*
- They are marked by an X
- Important for minimization (yet to come)