NAND Form

Networks and Embedded Software

Module 3.2.5 (optional)

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NAND Form (1)

- Instruction
  - Convert function to DNF (Disjunctive Normal Form)
    - Create the truth table
    - Read off the DNF
  - Convert DNF to NAND form
    - Double negation
    - De Morgan
    - NAND contraction
    - NOT elimination

Disjunctive Normal Form (1)

- Instruction
  - Handle only rows with a result of 1
  - Transform these rows into minterms
    - Connect all variables by conjunctions
    - Negate variable if they are 0
  - Connect all minterms by disjunctions

Disjunctive Normal Form (2)

- Example
  - Function
    - $\varphi(a,b) = a \oplus b$
  - Minterms
    - $m_0 = \neg a \wedge b$
    - $m_1 = a \wedge \neg b$
  - DNF
    - $\varphi(a,b) = (\neg a \wedge b) \vee (a \wedge \neg b)$

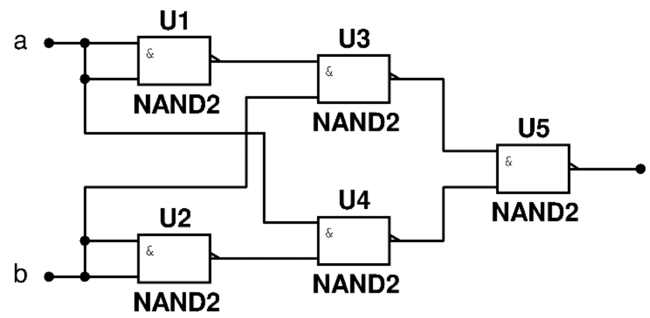
a	b	φ(a,b)	
0	0	0	
0	1	1	m_0
1	0	1	m_1
1	1	0	

NAND Form (2)

- Example (continued)
  - DNF
    - $\varphi(a,b) = (\neg a \wedge b) \vee (a \wedge \neg b)$
  - Double negation
    - $\varphi(a,b) = \neg\neg((\neg a \wedge b) \vee (a \wedge \neg b))$
  - De Morgan
    - $\varphi(a,b) = \neg\, (\neg(\neg a \wedge b) \wedge \neg(a \wedge \neg b))$

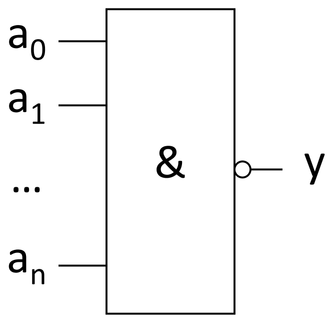
NAND Form (3)

- Example (finished)
  - NAND contraction
    - φ(a,b) = ¬ ((¬a|b) ∧ (a|¬b)) (inner terms)
    - φ(a,b) = (¬a|b) | (a|¬b) (outer terms)
  - NOT elimination
    - φ(a,b) = ((a|a)|b) | (a|(b|b))



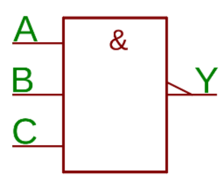
Circuit Symbol

- Compound NAND



$$|: \{0,1\}^n \rightarrow \{0,1\}$$

$$(a_0, a_1, \cdots) \mapsto \begin{cases} 0 \text{ if } a_0 = a_1 = \cdots = 1 \\ 1 \text{ else} \end{cases}$$



= ¬(A∧B∧C) ↮ A|B|C