Number Systems

Networks and Embedded Systems

First Grade Level

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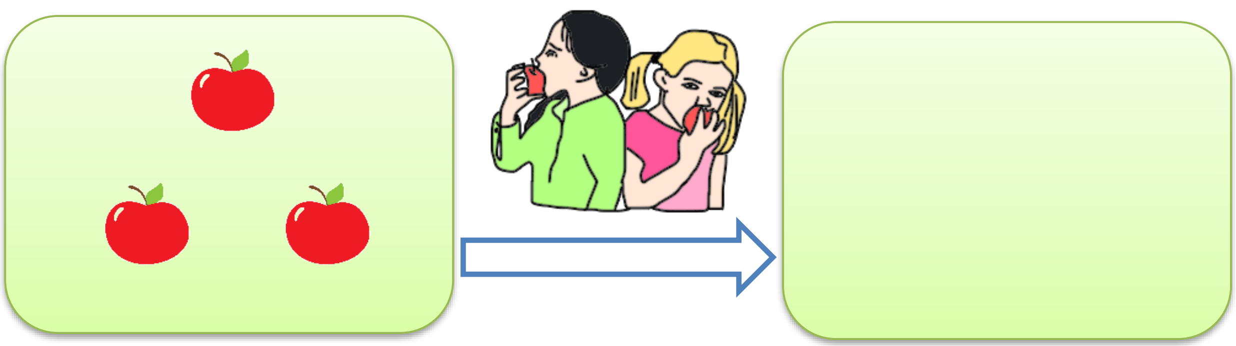
Number Systems (1)

- Numbers are abstract
  - They exist in mind, only
  - They have to be represented

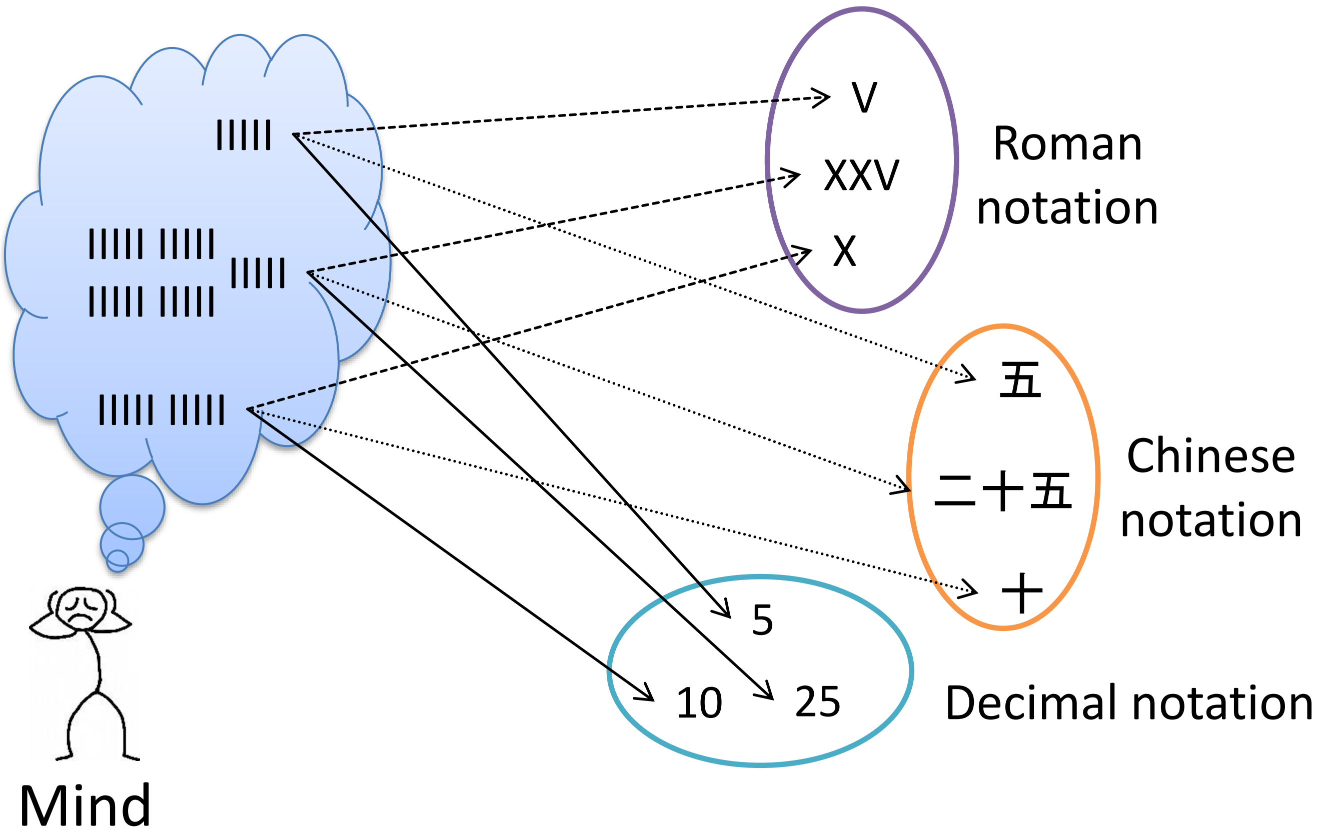


Three apples

The apples are eaten up.
What happened with the three?

Number Systems (2)

- Representation of Numbers



Decimal System (1)

- Our number system has 10 digits
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- The value of a digit depends on its place

thousand	hundred	ten	one
2	0	1	2

- The value of the place can be calculated

thousand	hundred	ten	one
10^3	10^2	10^1	10^0

Decimal System (2)

- What value has a series of digits?
  - Count the places starting with zero
  - Calculate the corresponding power of ten
  - Multiply the digit with the value of the place
  - Add up everything

Counting direction

Place	3	2	1	0
Value	10^3	10^2	10^1	10^0
	1000	100	10	1
Digit	2	0	1	2

Decimal System (3)

- What is the value of 2012?
  - $2012_{dec} = 2 \cdot 10^3 + 0 \cdot 10^2 + 1 \cdot 10^1 + 2 \cdot 10^0$
  - $2012_{dec} = 2 \cdot 1000 + 0 \cdot 100 + 1 \cdot 10 + 2 \cdot 1$
  - $2012_{dec} = 2012$
- We are familiar with the decimal system
- We know the value without calculation
- $2012_{\underline{dec}}$ → decimal number

Decimal System (4)

- What are the digits of a given value?
  - Divide by ten again and again
  - Note the remainder of the division
  - Stop if the result is zero
  - Read the remainders from the bottom up

Decimal System (5)

- What are the digits of 2012?
  - $2012 \div 10 = 201$ remainder 2
  - $201 \div 10 = 20$ remainder 1
  - $20 \div 10 = 2$ remainder 0
  - $2 \div 10 = 0$ remainder 2

reading direction

Calculation stops here

- The result is 2012_{dec}
- We know the digits without calculation
  - We are familiar with the decimal system

Decimal System (6)

- Let's assume a certain number of digits
  - How many numbers can be represented?
    - Ten to the power of number of digits
  - What is the largest number?
    - Put the largest digit on every position
    - Count of all possible numbers minus one
- Suppose there are four digits
  - Count of numbers: 10^4 = 10000 (0 … 9999)
  - Largest number: 9999 or 10000-1 = 9999

Largest digit at every position

Count of numbers minus one

Hexadecimal System (1)

- The hexadecimal system has 16 digits
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- The characters represent the values

A	B	C	D	E	F
10	11	12	13	14	15

- The value of the places can be calculated

Place	3	2	1	0
Value	16^3	16^2	16^1	16^0

Hexadecimal System (2)

- What is the value of 2012_{hex}?
  - $2012_{hex} = 2 \cdot 16^3 + 0 \cdot 16^2 + 1 \cdot 16^1 + 2 \cdot 16^0$
  - $2012_{hex} = 2 \cdot 4096 + 0 \cdot 256 + 1 \cdot 16 + 2 \cdot 1$
  - $2012_{hex} = 8210$
- The hexadecimal system is strange
- We have to calculate the value
- $2012_{\underline{hex}} \rightarrow$ hexadecimal number

Hexadecimal System (3)

- What are the digits of 2012?
  - $2012 \div 16 = 125$ remainder 12 (C)
  - $125 \div 16 = 7$ remainder 13 (D)
  - $7 \div 16 = 0$ remainder 7

Calculation stops here

reading direction

- The result is $7DC_{hex}$
- We do not know the digits without calculation
  - We are not familiar with the hexadecimal system

Hexadecimal System (4)

- Suppose there are four hexadecimal digits
  - How many numbers are there in total?
    - $16^4 = 65536$
  - What is the largest number?
    - $FFFF_{hex}$ At all positions the largest digit
    - 65535 Count of numbers minus one

Hexadecimal System (5)

- Summary
  - From representation to value
    - $7DC_{hex} \rightarrow 2012$ Hexadecimal representation → decimal value
    - Multiplication with place values
  - From value to representation
    - $2012 \rightarrow 7DC_{hex}$ Decimal value → hexadecimal representation
    - Division by base

Binary System (1)

- The binary system has 2 digits
  - 0, 1
- What is the value of 1011_{bin}?
  - $1011_{bin} = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$
  - $1011_{bin} = 1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 1 \cdot 1$
  - $1011_{bin} = 11$

Binary System (2)

- What are the digits of 11?
  - $11 \div 2 = 5$ remainder 1
  - $5 \div 2 = 2$ remainder 1
  - $2 \div 2 = 1$ remainder 0
  - $1 \div 2 = 0$ remainder 1

reading direction

Calculation stops here

- The result is 1011_{bin}

Binary System (3)

- Suppose there are four binary digits
  - How many numbers are there in total?
    - $2^4 = 16$
  - What is the largest number?
    - 1111_{bin}
    - 15

At all positions the largest digit

Count of numbers minus one

  - Common terms
    - One binary digit = 1 bit
    - Eight binary digits = 1 byte
    - Series of binary digits = bit string

Number Systems (3)

- Number systems are universal
  - They work with any number of digits
- Examples
  - Octal system
    - 8 digits
    - Old representation for data bytes
  - Base64
    - 64 digits
    - Used to transfer binary data by email

Number Systems (4)

- Number systems are laborious
  - One has to calculate a lot
- Sometimes there is a simpler method
  - Binary → decimal
  - Decimal → binary
  - Binary → hexadecimal
  - Hexadecimal → binary

Fast Conversions (1)

- From binary to decimal
  - Note … 8 4 2 1 over the positions
    - Start with one beginning at the right hand side
    - Go ahead to the left and double the number
  - Add the values of all positions with a one

Fast Conversions (2)

- What is the value of 10110011_{bin}?

128	64	32	16	8	4	2	1
1	0	1	1	0	0	1	1

  – 128 + 32 + 16 + 2 + 1 = 179

- The value of 10110011_{bin} is 179

Fast Conversions (3)

- From decimal to binary
  - Double one until it is larger than the value
  - Try to subtract the half of this number
    - Note 1 if it is possible
    - Note 0 if it is not possible
  - Continue until one is reached again
  - The digits noted are the binary number

Fast Conversions (4)

- What are the digits of 179?
  - Double one until it is larger than the value
    - 1 2 4 8 16 32 64 128 256.
  - Try to subtract the half of this number

179	51	51	19	3	3	3	1
128	64	32	16	8	4	2	1
1	0	1	1	0	0	1	1

- 179 is 10110011_{bin}

Fast Conversions (5)

- Binary ↔ Hexadecimal
  - Create a table
    - On the left there are the hexadecimal digits
    - On the right there are the corresponding bits

0	0000	**4**	0100	**8**	1000	**C**	1100
1	0001	**5**	0101	**9**	1001	**D**	1101
2	0010	**6**	0110	**A**	1010	**E**	1110
3	0011	**7**	0111	**B**	1011	**F**	1111

Fast Conversions (6)

- Hexadecimal → Binary
  - Proceed digit by digit
  - Look up the bit pattern in the table
- What are $A7_{hex}$ and BC_{hex} as binary numbers
  - $A7_{hex} \rightarrow 1010\ 0111_{bin}$
  - $BC_{hex} \rightarrow 1011\ 1100_{bin}$

Fast Conversions (7)

- Binary → Hexadecimal
  - Starting from the right make groups of four
  - Add zeros if necessary
  - Look up the groups of four in the table
- Convert 110100_{bin} and 10111100_{bin}
  - $110100_{bin} \rightarrow 0011\ 0100_{bin} \rightarrow 34_{hex}$
  - $10111100_{bin} \rightarrow \underbrace{1011}_{2^{nd}\text{ group}}\ \underbrace{1100}_{1^{st}\text{ group}}{}_{bin} \rightarrow BC_{hex}$

Number Systems (5)

- Decimal and binary do not fit well
  - Ten is no power of two
- Hexadecimal und binary fit quite well
  - Sixteen is a power of two (2^4)
  - Four bits are exactly one hexadecimal digit
  - One hexadecimal digit is exactly four bits
- Bit strings are usually written as hexadecimals
  - The bit strings by itself would be much too long

Number Systems (6)

- Application of the hexadecimal system
  - Hex editor
    - A hex editor display the content of a file as a sequence of hexadecimal numbers
    - The content of the file can be changed by changing the hexadecimal numbers

```
89 50 4E 47 0D 0A 1A 0A 00 00 00 0D 49 48 44 52   ‰PNG........IHDR
00 00 00 01 00 00 00 01 08 02 00 00 00 90 77 53   ..............wS
DE 00 00 00 0E 49 44 41 54 78 DA 62 F8 CF C0 00   Þ....IDATxÚbøÏÀ.
10 60 00 03 01 01 00 66 FD 9F 24 00 00 00 00 49   .`.....fýŸ$....I
45 4E 44 AE 42 60 82                              END®B`‚
```

Character Encoding (1)

- ASCII (American Standard Code for Information Interchange)

ASCII		Lower Hex Digit															
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**A**	**B**	**C**	**D**	**E**	**F**
Higher Hex Digit	**0**	NUL	SOH	STX	ETX	EOF	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
	1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
	2		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
	3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
	4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
	6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
	7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

Character Encoding (2)

- Characters are encoded as bit strings
- The code is represented as hexadecimal
- The code can be looked up in the table
- What code have \$ and n?
  - \$ → row 2, column 4 → 24_{hex} → $0010\ 0100_{bin}$
  - n → row 6, column E → $6E_{hex}$ → $0110\ 1110_{bin}$

higher hex digit

lower hex digit

  - Attention: Do not confuse higher und lower hex digit