

Truth Tables

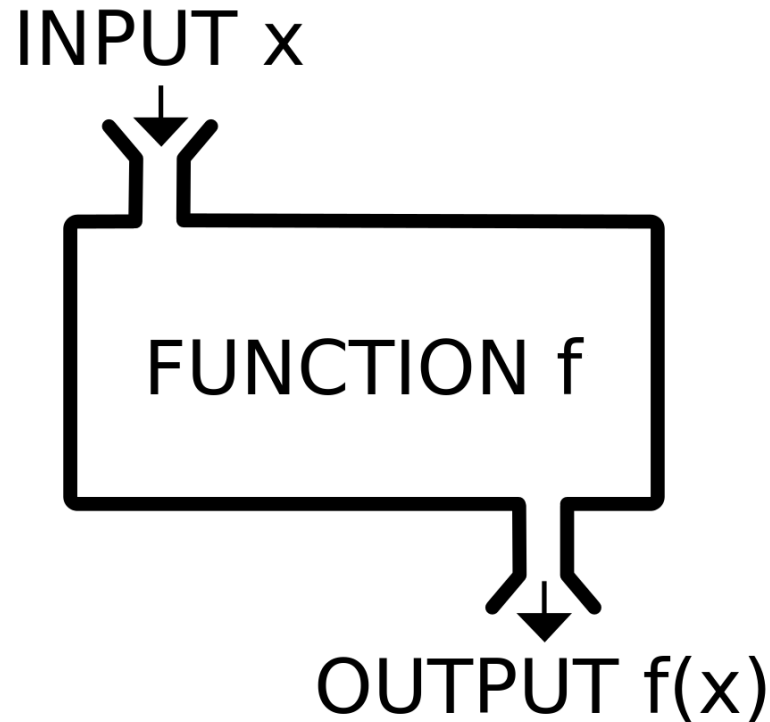
Networks and Embedded Systems

First Grade Level

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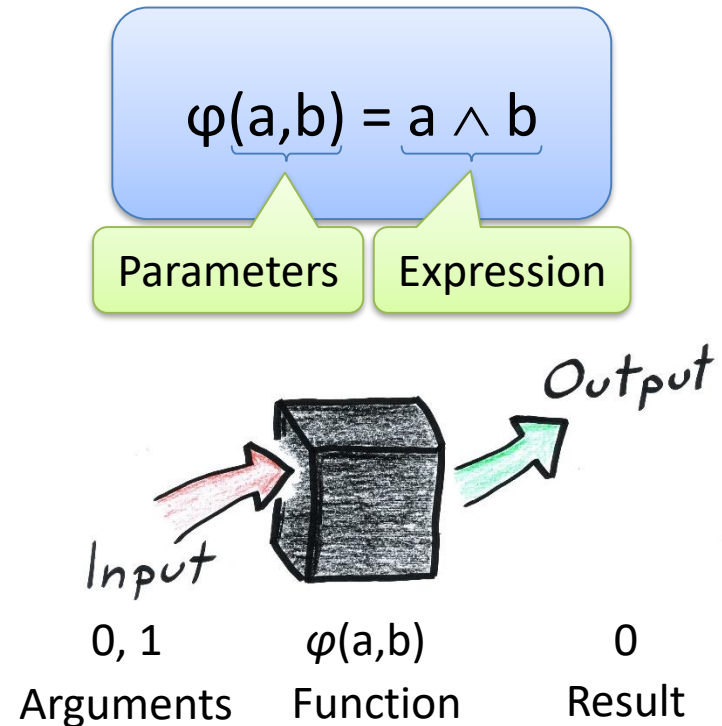
Truth Tables (1)

- Functions
 - Process input
 - Produce output
 - They have
 - Parameters
 - Input
 - They return
 - Results
 - Output



Truth Tables (2)

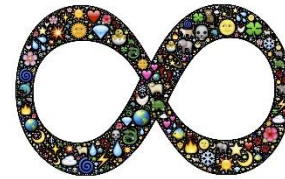
- Logical Functions
 - Are expression of
 - Parameters (a, b, c, \dots)
 - Operators ($\neg, \wedge, \vee, \dots$)
 - Functions ($\varphi, \psi, \chi, \dots$)
 - Alternative terms
 - Logic function
 - Boolean function
 - Switching function



Truth Tables (3)

- Logical Functions (continued)
 - Arithmetic and truth functions are different

- Arithmetic functions are infinite
- Truth functions are finite



$$f(1,1) = 2$$

$$f(1,2) = 3$$

$$f(1,3) = 4$$

$$f(1,4) = 5$$

...

Arguments never end

Arithmetic function $f(x,y) = x+y$

$$\varphi(0,0) = 0$$

$$\varphi(0,1) = 0$$

$$\varphi(1,0) = 0$$

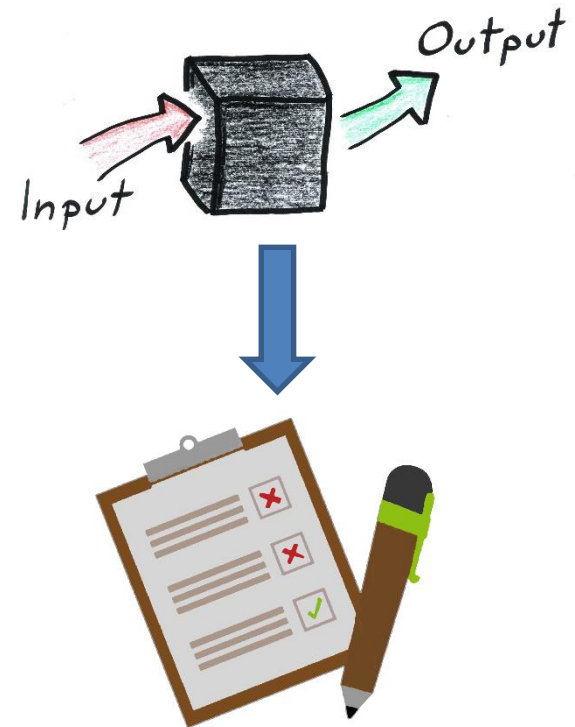
$$\varphi(1,1) = 1$$

No arguments any more

Truth function $\varphi(a,b) = a \wedge b$

Truth Tables (4)

- Logical Functions (continued)
 - A truth table can be made
 - All possible inputs (arguments)
 - All possible outputs (results)
 - Alternative terms
 - Truth table
 - Switching table
 - State table



Truth Tables (5)

- Logical Functions (finished)

- Example

- $\varphi(a,b) = a \wedge b$



a	b	$\varphi(a,b)$
0	0	0
0	1	0
1	0	0
1	1	1

- Description

- Two parameters

- a, b

- Four arguments

- (0,0), (0,1), (1,0), (1,1)

- One result

- 0 or 1

Truth Tables (6)

- Examples

- Functions with one parameter: $\varphi(a)$

- Functions with two parameters: $\varphi(a,b)$

a	$\varphi(a)$
0	...
1	...

a	b	$\varphi(a,b)$
0	0	...
0	1	...
1	0	...
1	1	...

Truth Tables (7)

- Construction
 - Number of columns
 - Number of parameters: n
 - Plus number of results
 - Number of rows
 - Number of arguments: 2^n
 - First column
 - Fifty-fifty: $\frac{1}{2}$ column 0, $\frac{1}{2}$ column 1

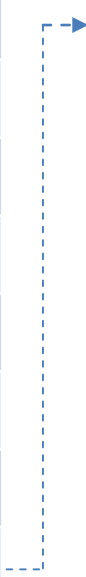
Truth Tables (8)

- Construction (continued)
 - Second column
 - Fifty-fifty but twice as fast: $\frac{1}{4} 0, \frac{1}{4} 1, \frac{1}{4} 0, \frac{1}{4} 1$
 - And so on ...
 - Checklist
 - First rows starts with: 0 0 0 0 ...
 - Last rows ends with: 1 1 1 1 ...
 - Last column is alternating: 0 1 0 1 0 1 ...
 - Sequence of rows: binary natural numbers (0 1 2 3 ...)

Truth Tables (9)

- Functions with four parameters: $\varphi(a,b,c,d)$

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1



a	b	c	d
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

continued on the right

Truth Tables (10)

- Logical Equivalence
 - Truth tables are identical
 - $\varphi \leftrightarrow \psi$

a	b	$\varphi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

a	b	$\psi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

Truth Tables (11)

- Creation (first method)
 - Mathematician's method
 - Term by term
 - Terms: compose the expression of a function
 - Functions: span a tree of terms

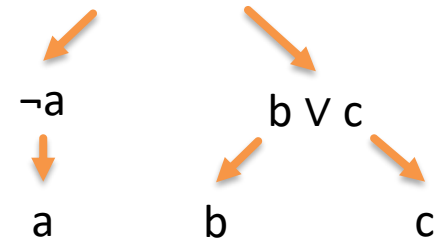


$$\varphi(a,b,c) = \underbrace{\neg a}_{1^{\text{st}} \text{ term}} \wedge \underbrace{(b \vee c)}_{2^{\text{nd}} \text{ term}}$$

$\underbrace{\hspace{10em}}_{3^{\text{rd}} \text{ term}}$



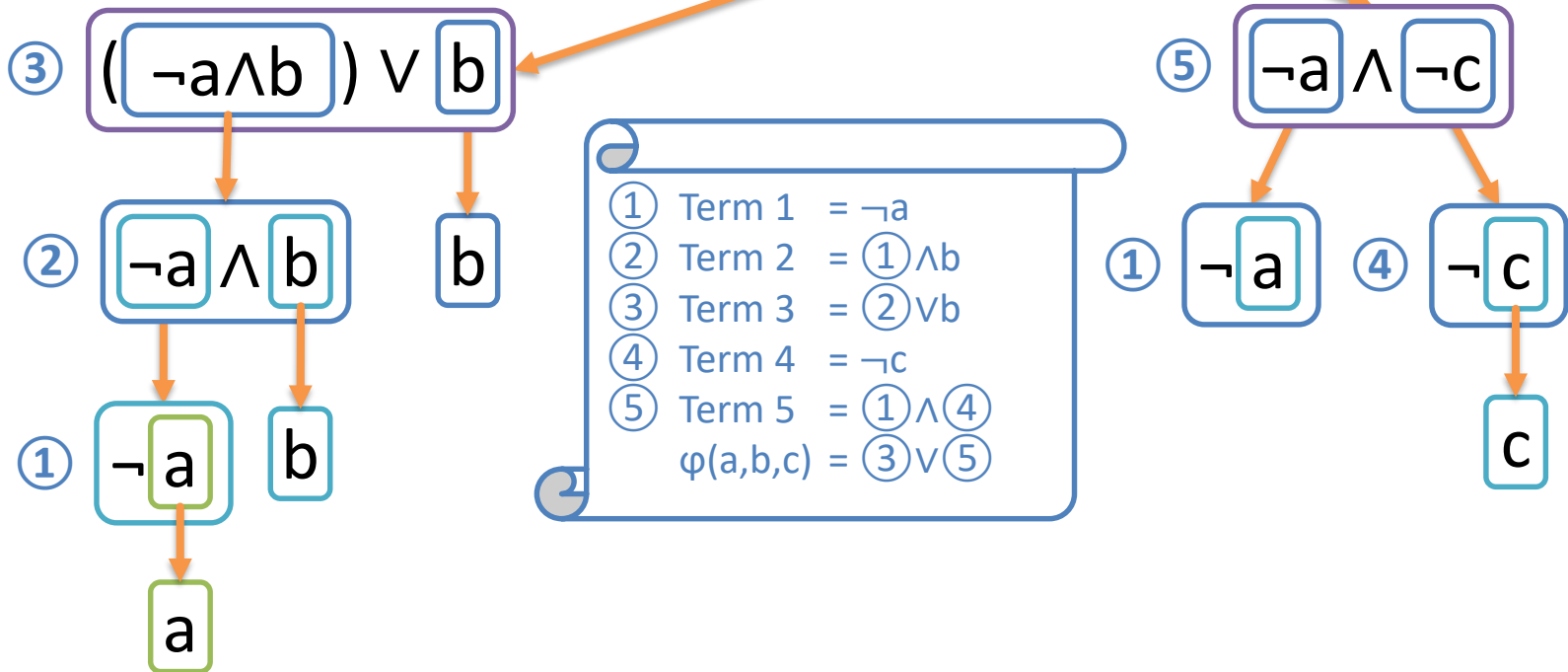
$$\varphi(a,b,c) = \neg a \wedge (b \vee c)$$



Truth Tables (12)

- Creation (first method, continued)

– Example: $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$



Truth Tables (13)

- Creation (first method, finished)
 - Example: $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

			①	②	③	④	⑤	$\varphi(a,b,c)$
a	b	c	$\neg a$	① \wedge b	② \vee b	$\neg c$	① \wedge ④	③ \vee ⑤
0	0	0	1	0	0	1	1	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	1	1	0	1
1	1	1	0	0	1	0	0	1

Truth Tables (14)

- Creation (second method)
 - Philosopher's method
 - Put each symbol in a column
 - Fill in the arguments
 - Perform the operations
 - Fill the column with the results
 - The last column shows the result of the truth function.



a	b	a	\wedge	b
0	0	0	0	0
0	1	0	0	1
1	0	1	0	0
1	1	1	1	1

Truth Tables (15)

- Creation (second method, continued)

– Example: $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

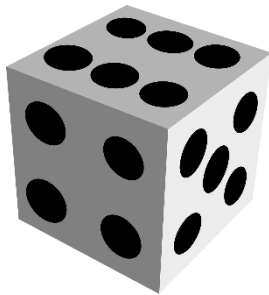
Sequence				2	1	4	3		6	5		12		8	7	11	10	9		
a	b	c	((\neg	a	\wedge	b)	\vee	b)	\vee	(\neg	a	\wedge	\neg	c)
0	0	0			1	0	0	0		0	0		1		1	0	1	1	0	
0	0	1			1	0	0	0		0	0		0		1	0	0	0	1	
0	1	0			1	0	1	1		1	1		1		1	0	1	1	0	
0	1	1			1	0	1	1		1	1		1		1	0	0	0	1	
1	0	0			0	1	0	0		0	0		0		0	1	0	1	0	
1	0	1			0	1	0	0		0	0		0		0	1	0	0	1	
1	1	0			0	1	0	1		1	1		1		0	1	0	1	0	
1	1	1			0	1	0	1		1	1		1		0	1	0	0	1	







Truth Tables (16)

- Don't-Care Terms
 - Truth tables can be incomplete
 - Length of truth table is fixed: 2^n
 - Unused rows are called don't-care terms
 - They must not be omitted
 - They are marked by X

Truth Tables (17)

- Don't-Care Terms (continued)
 - Example: Which face of a dice has six pips?



n	a	b	c	$\varphi(a,b,c)$	Face
0	0	0	0	X	invalid
1	0	0	1	0	
2	0	1	0	0	
3	0	1	1	0	
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	1	
7	1	1	1	X	invalid

Truth Tables (18)

- Conclusion
 - Truth tables describe logic functions
 - One function has exactly one truth table
 - Truth tables are not unique
 - Many functions have the same truth table
 - These functions are logically equivalent
 - Some of these functions can easily be found
 - Truth table \rightarrow logic function \rightarrow digital circuit