Combinatorial Circuits

Mechanical and Electrical Engineering

Second Grade Level

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Combinatorial Circuits (1)

- Design of Combinatorial Circuits
  - Problem: Please add two bits
    - Hints
      - If you add two numbers a carry can occur
      - You need not only determine the result but also the carry
      - On the other hand there can already be a carry when you add two numbers
      - You have to handle this carry, too
      - This type of circuit is called full adder
    - Mathematical description of a full adder
      - $\{0,1\}\times\{0,1\}\times\{0,1\} \longmapsto \{0,1\}\times\{0,1\}$
        $(a,b,c_i) \longrightarrow (c_o,r)$
        c_i: carry in; c_o: carry out; r: LSB of a+b (left most bit)

Or for short:
$\{0,1\}^3 \longmapsto \{0,1\}^2$

Combinatorial Circuits (2)

- Design of Combinatorial Circuits (continued)
  - Example of a Binary Addition

Bit position	3	2	1	0
1st Number (7)	0	1	1	1
2nd Number (5)	0	1	0	1
Carry	1	1	1	0
Result (12)	1	1	0	0

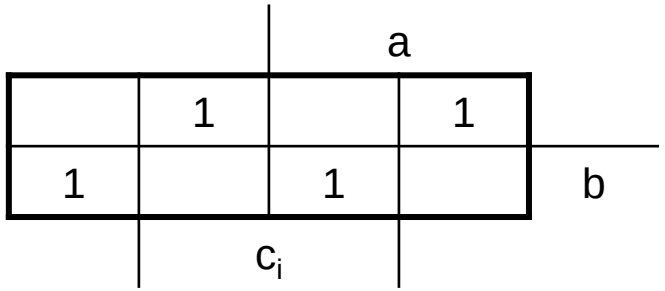
Combinatorial Circuits (3)

- Design of Combinatorial Circuits (continued)
  - Truth table of a one bit full adder

a	b	c_i	c_o	y
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Combinatorial Circuits (4)

- Design of Combinatorial Circuits (continued)
  - Switching function $y(a,b,c_i)$



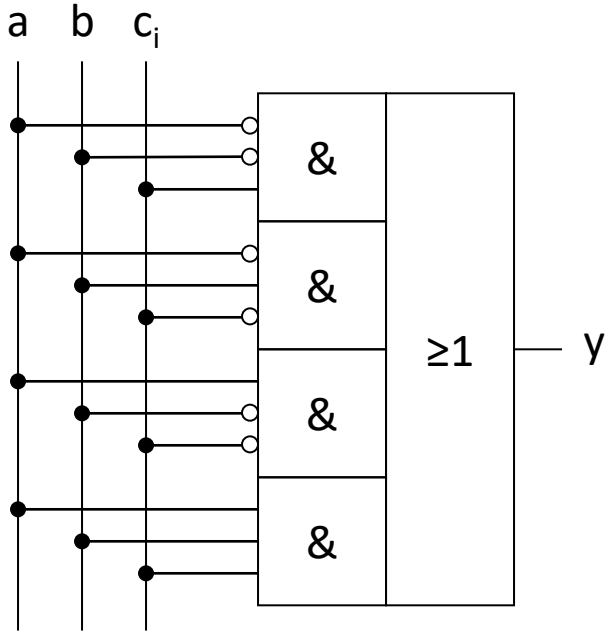
$t_0 = \neg a \wedge \neg b \wedge c_i$

$t_1 = \neg a \wedge b \wedge \neg c_i$

$t_2 = a \wedge \neg b \wedge \neg c_i$

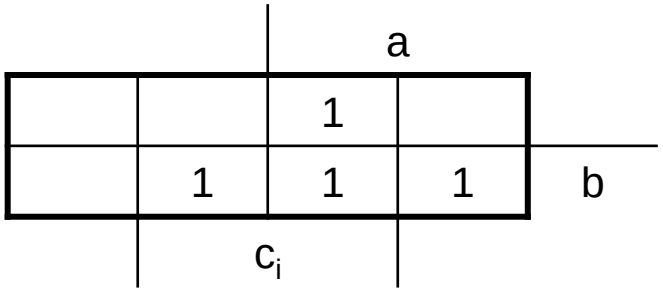
$t_3 = a \wedge b \wedge c_i$

$y = t_0 \vee t_1 \vee t_2 \vee t_3$



Combinatorial Circuits (5)

- Design of Combinatorial Circuits (continued)
  - Switching function $c_o(a,b,c_i)$

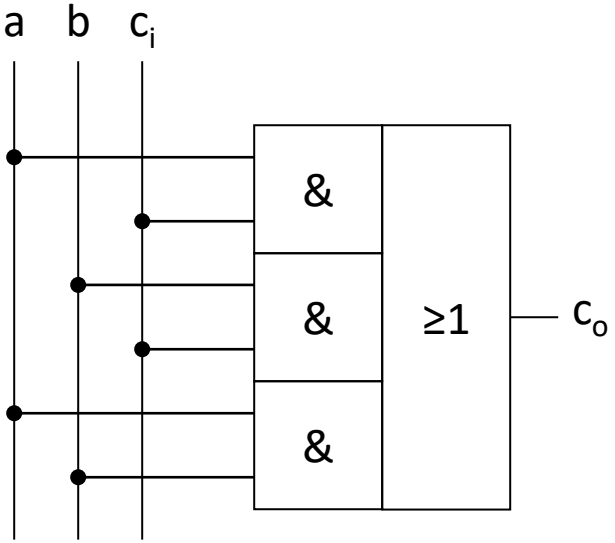


$t_0 = a \wedge c_i$

$t_1 = b \wedge c_i$

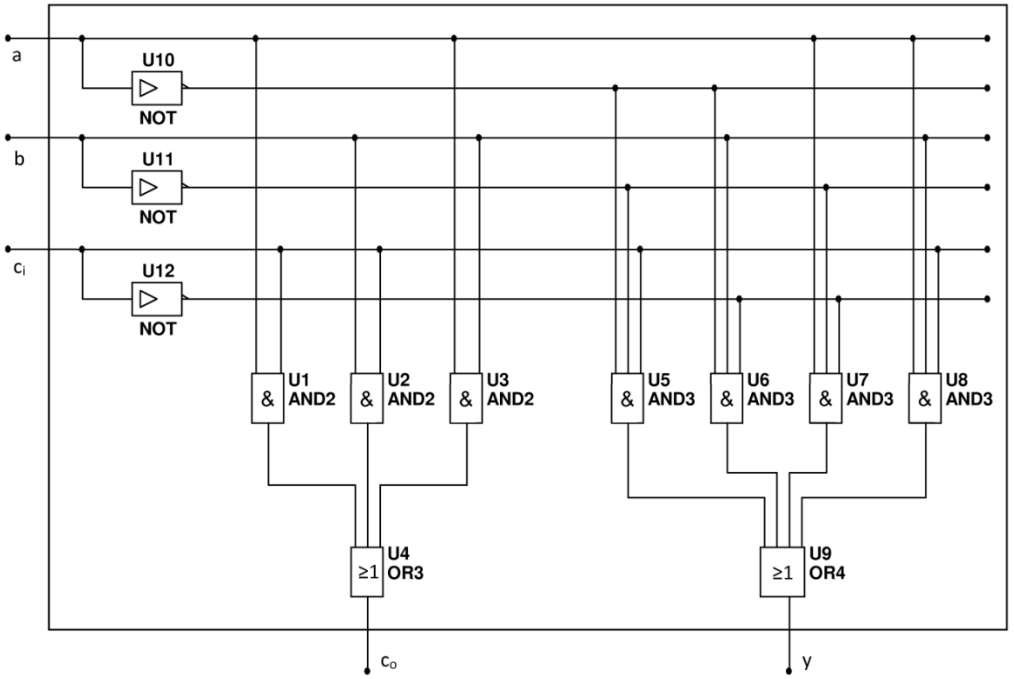
$t_2 = a \wedge b$

$c_o = t_0 \vee t_1 \vee t_2$

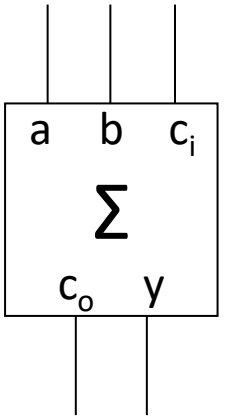


Combinatorial Circuits (6)

- Design of Combinatorial Circuits (continued)
  - Both functions integrated in a circuit

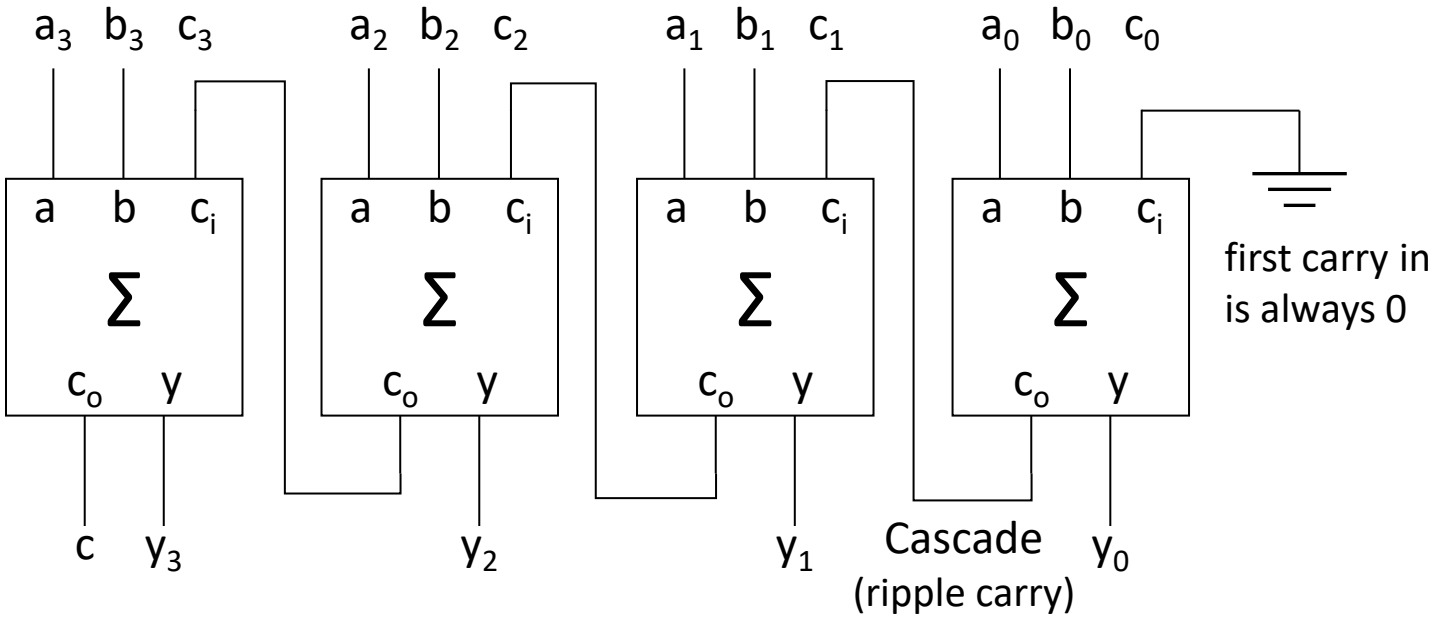


One-Bit Full Adder



Combinatorial Circuits (7)

- Design of Combinatorial Circuits (continued)
  - Four-bit Full Adder
    - Cascade of four one-bit full adders



Combinatorial Circuits (8)

- Design of Combinatorial Circuits (finished)
  - Integrated four bit half adder