Capacitors and Inductors

Electrical Engineering

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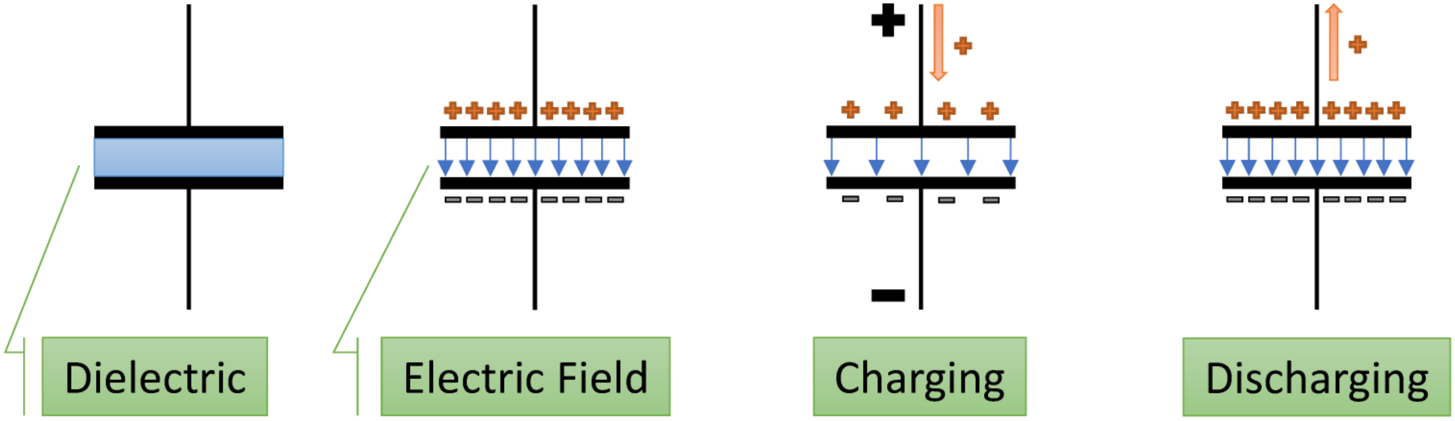
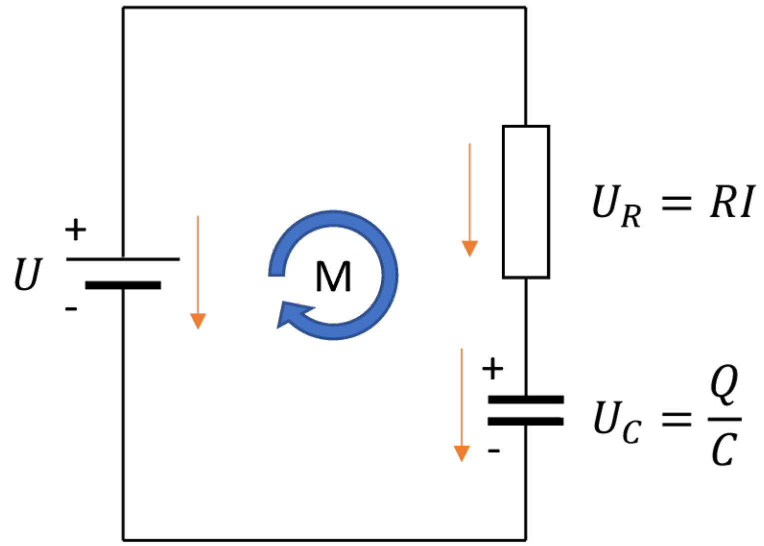
Capacitors (1)

- Basic Features
  - Accumulates electric charges on two surfaces
    - The surfaces are insulated from each other
  - Stores electrical energy in an electric field
  - It can be charged and discharged



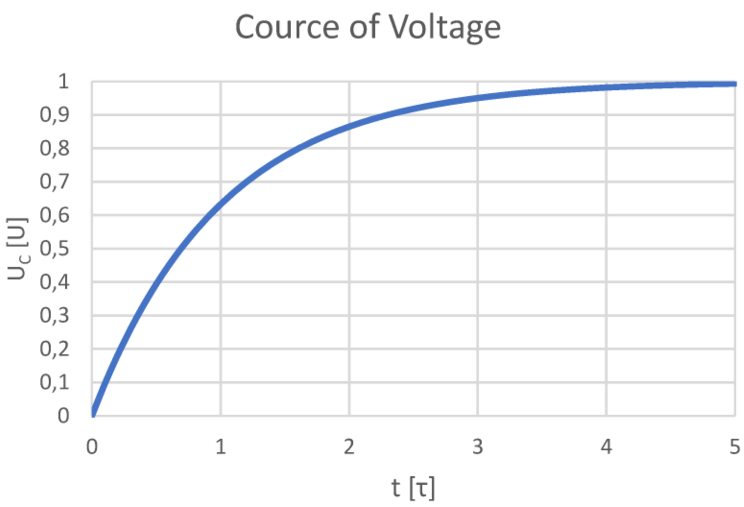
Capacitors (2)

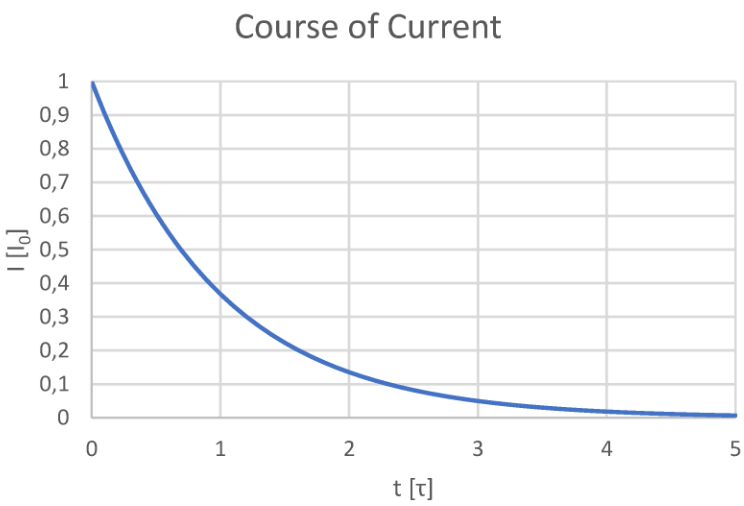
- Charging
  - Basic Relations
    - $R = \frac{U}{I}$
    - $I = \frac{dQ}{dt} = \dot{Q}$
    - $Q = C \cdot U$
  - Kirchhoff's circuit law
    - $U_R + U_C - U = 0$
    - $R\dot{Q} + \frac{Q}{C} - U = 0$
  - Differential Equation
    - $\dot{Q} = \frac{1}{R}U - \frac{1}{RC}Q$



Capacitors (3)

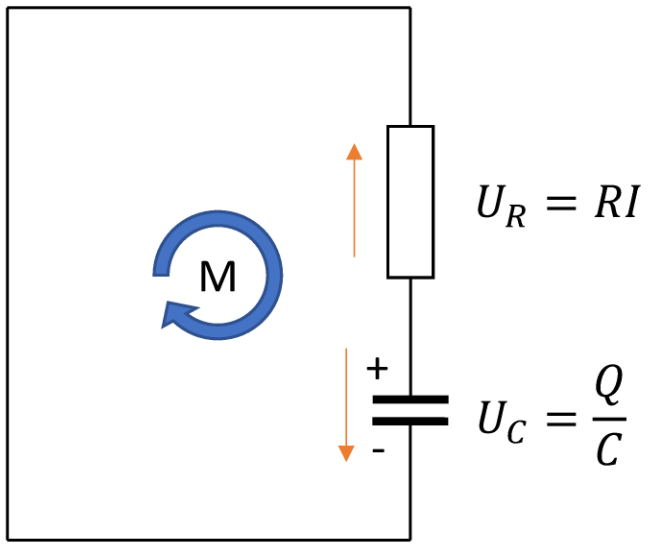
- Charging (continued)
  - Solution of the equation
    - $Q = CU - CUe^{-\frac{1}{RC}t}$
    - $\tau = RC$ (time constant)
  - Course of the voltage
    - $U_C = \frac{Q}{C}$
    - $U_C = U\left(1 - e^{-\frac{t}{\tau}}\right)$
  - Course of the current
    - $I = \dot{Q}$
    - $I = \frac{U}{R}e^{-\frac{t}{\tau}}$





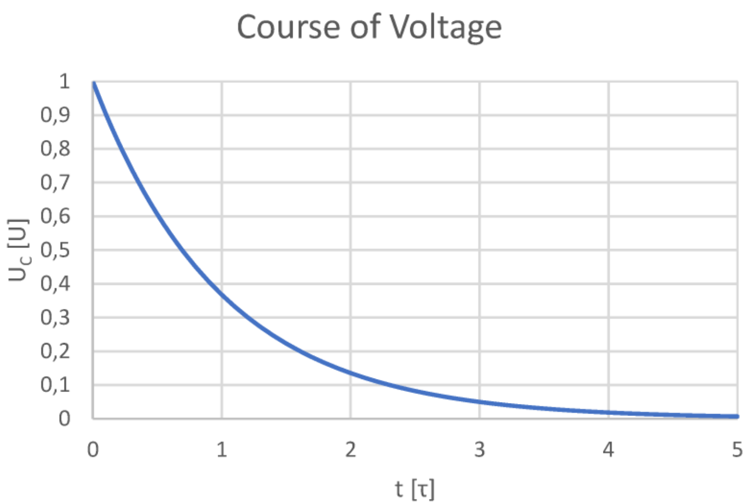
Capacitors (4)

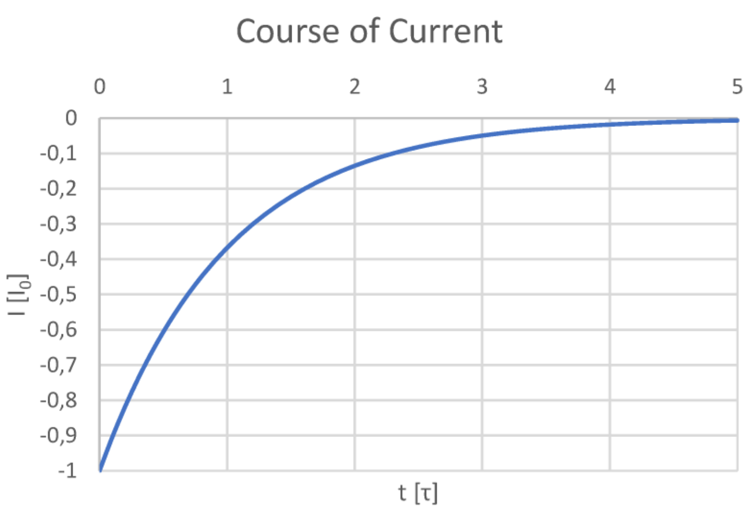
- Discharging
  - Basic Relations
    - $R = \frac{U}{I}$
    - $I = \frac{dQ}{dt} = \dot{Q}$
    - $Q = C \cdot U$
  - Kirchhoff's circuit law
    - $-U_R + U_C = 0$
    - $-R\dot{Q} + \frac{Q}{C} = 0$
  - Differential Equation
    - $\dot{Q} = -\frac{1}{RC}Q$



Capacitors (5)

- Discharging (continued)
  - Solution of the equation
    - $Q = CUe^{-\frac{1}{RC}t}$
    - $\tau = RC$ (time constant)
  - Course of the voltage
    - $U_C = \frac{Q}{C}$
    - $U_C = U\left(e^{-\frac{t}{\tau}}\right)$
  - Course of the current
    - $I = \dot{Q}$
    - $I = -\frac{U}{R}e^{-\frac{t}{\tau}}$





Capacitors (6)

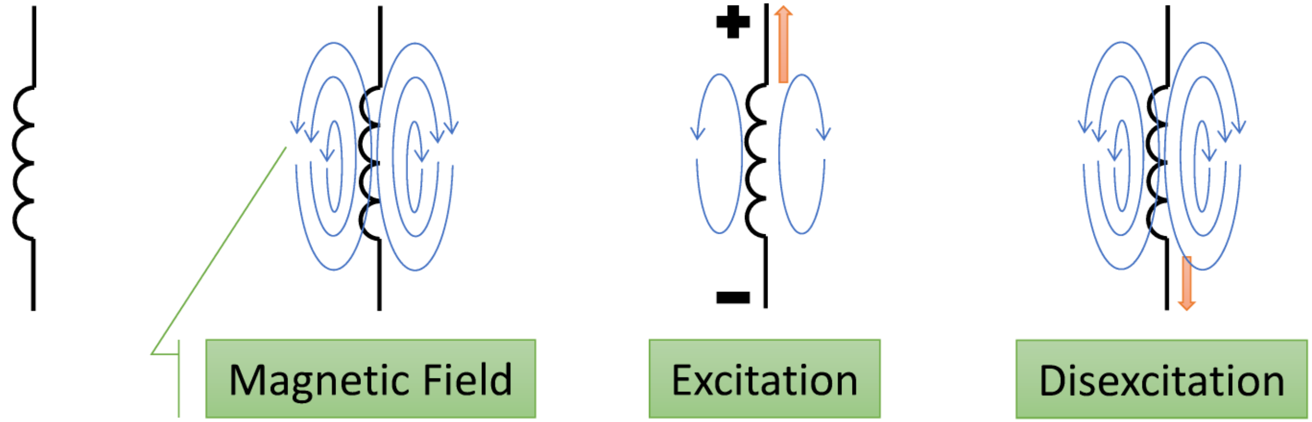
- Time Constant
  - $\tau = RC$
  - Half-value Time
    - $e^{-\frac{t_h}{\tau}} = \frac{1}{2}$
    - $t_h = \tau \ln 2$
    - $t_h = 0.69\tau$
  - Full Charge Time
    - $t_f = 5\tau$
    - $e^{-5} = 0.0067$
    - $1 - e^{-5} = 0.993$

τ	0	1	2	3	4	5
Charge	0%	63%	86%	95%	98%	99%
Discharge	100%	37%	14%	5%	2%	1%

τ	0.0	0.1	0.2	0.3	0.4	0.5
Charge	0%	10%	18%	26%	33%	39%
Discharge	100%	90%	82%	74%	67%	61%

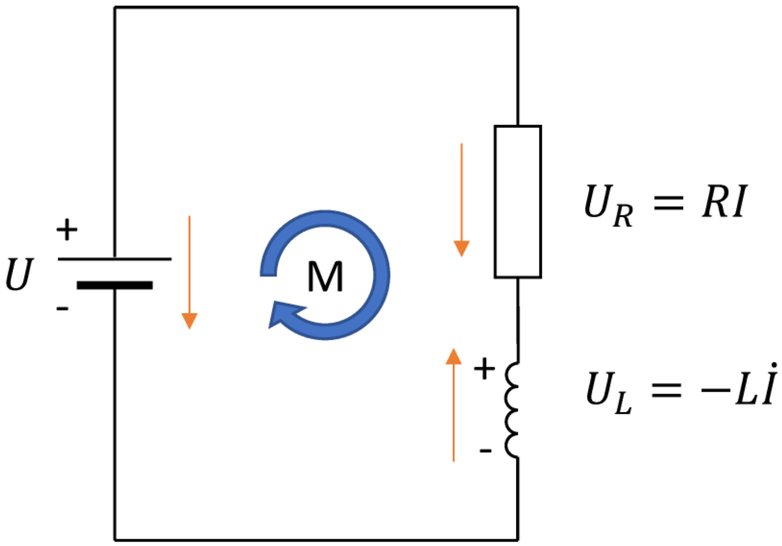
Inductors (1)

- Basic Features
  - Stores electric energy when current flows through it
  - The energy gets stored in a magnetic field
  - A magnetic field can be excited and disexcited
  - The induced current opposes the change of the field



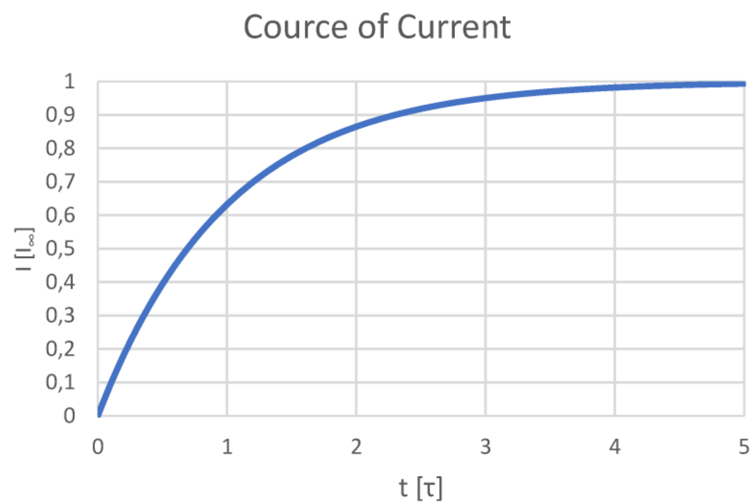
Inductors (2)

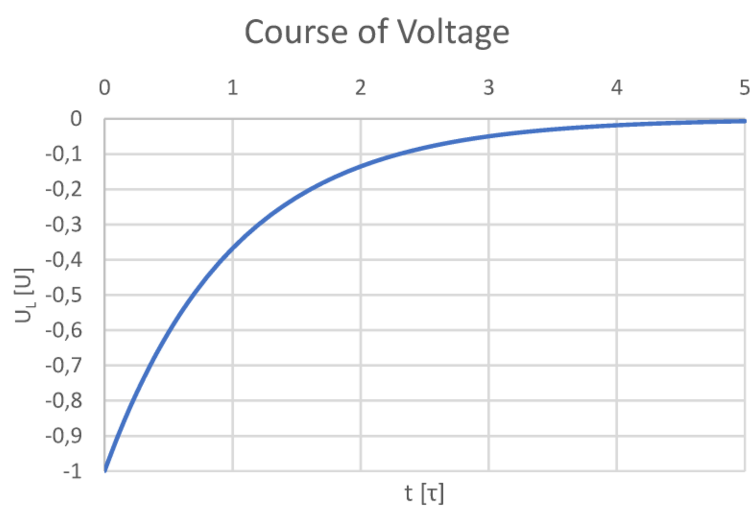
- Excitation
  - Basic Relations
    - $R = \frac{U}{I}$
    - $U_L = -L\frac{dI}{dt} = -L\dot{I}$
  - Kirchhoff's circuit law
    - $U_R - U_L - U = 0$
    - $RI + L\dot{I} - U = 0$
  - Differential Equation
    - $\dot{I} = \frac{1}{L}U - \frac{R}{L}I$



Inductors (3)

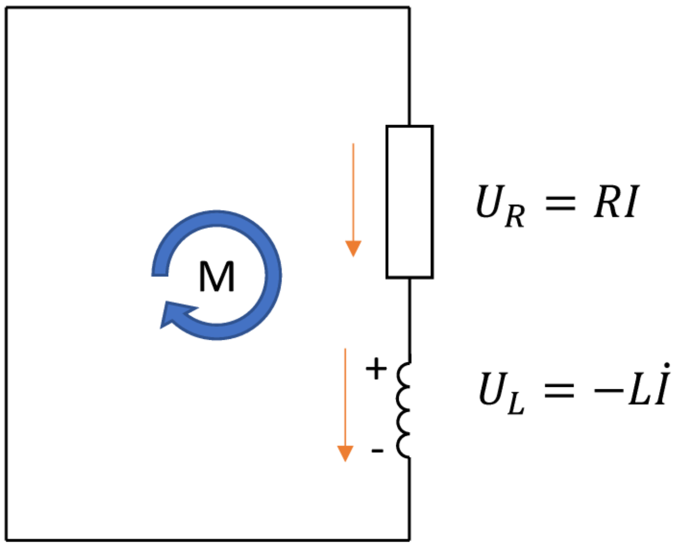
- Excitation (continued)
  - Solution of the equation
    - $I = \frac{U}{R}\left(1 - e^{-\frac{R}{L}t}\right)$
    - $\tau = \frac{L}{R}$ (time constant)
  - Course of the current
    - $I = \frac{U}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$
  - Course of the voltage
    - $U_L = -L\dot{I}$
    - $U_L = -Ue^{-\frac{t}{\tau}}$





Inductors (4)

- Disexcitation
  - Basic Relations
    - $R = \frac{U}{I}$
    - $U_L = -L\frac{dI}{dt} = -L\dot{I}$
  - Kirchhoff's circuit law
    - $U_R + U_L = 0$
    - $RI - L\dot{I} = 0$
  - Differential Equation
    - $\dot{I} = \frac{R}{L} I$



Capacitors (5)

- Disexcitation (continued)
  - Solution of the equation
    - $I = \frac{U}{R} e^{-\frac{R}{L}t}$
    - $\tau = \frac{L}{R}$ (time constant)
  - Course of the current
    - $I = \frac{U}{R} e^{-\frac{t}{\tau}}$
  - Course of the voltage
    - $U_L = -L\dot{I}$
    - $U_L = U e^{-\frac{t}{\tau}}$