Logical Operators

Digital Electronics

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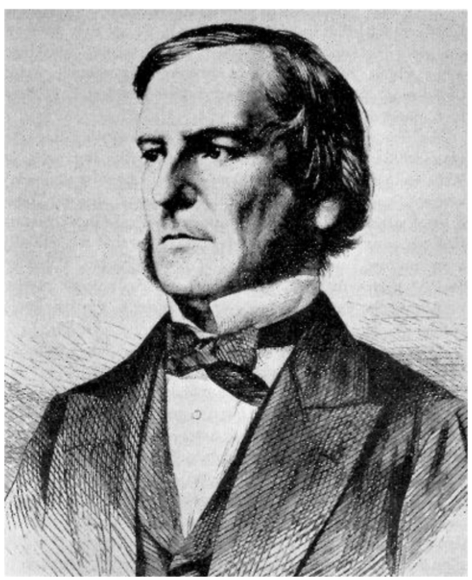
Boolean Algebra (1)

- George Boole
  - British mathematician and philosopher
  - Mathematical foundation of computer science



  - * 2 Nov 1815 in England
  - † 8 Dec 1864 in Ireland

Boolean Algebra (2)

- Calculating with Truth
  - True → 1
  - False → 0

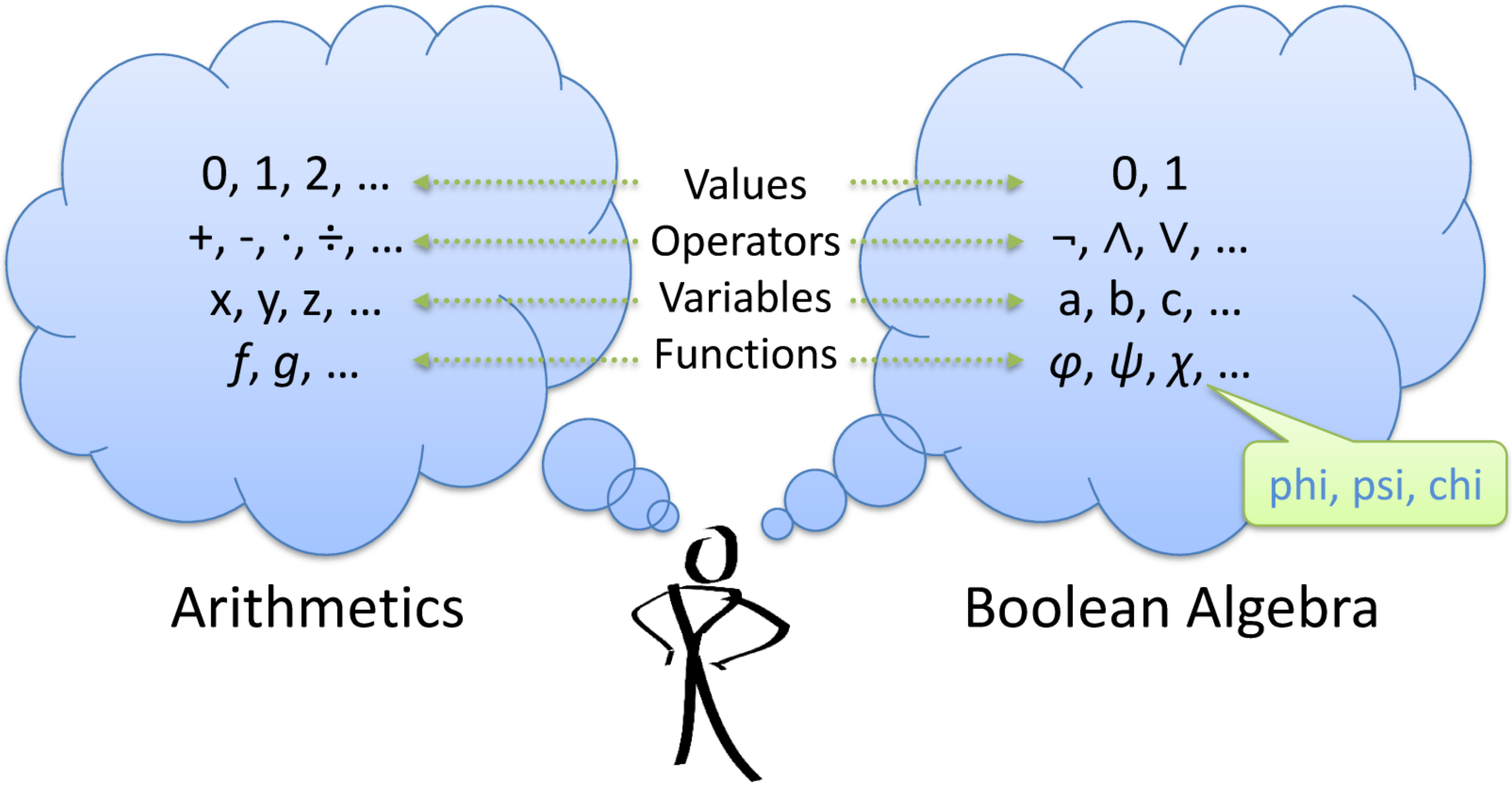
Something that can be true or false.
Calculating with truth is difficult.

$$1+1 = 2$$
$$0-1 = -1$$
$$1 \div 0 = \bot$$

A new kind of mathematics is necessary

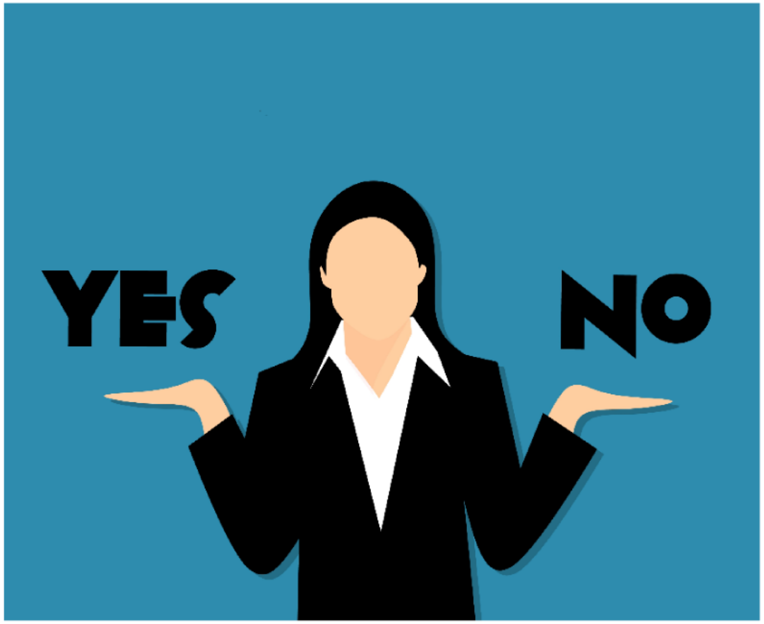
Boolean Algebra (3)

- A new kind of mathematics



Logical Operators (1)

- Operate on logical values
  - True/False, On/Off, High/Low, 1/0
- Alternative terms
  - Logical operator
    - Logical connective
    - Boolean operator
  - Logical value
    - Truth value
    - Boolean value

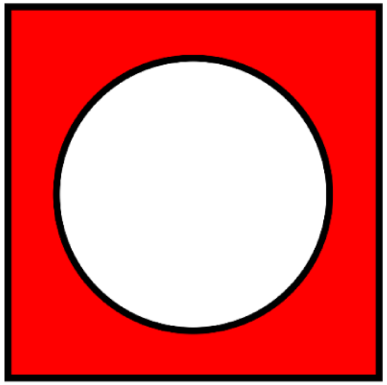


Logical Operators (2)

- Negation
  - Symbol: ¬ (NOT, sometimes ¬A → Ā)
  - Meaning: logical not (contrary of …)
  - Definition:

a	¬a
0	1
1	0

Truth Table

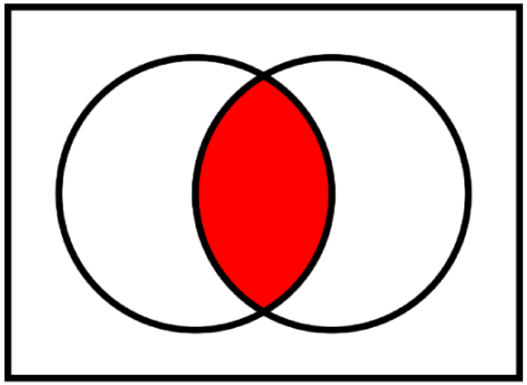


Venn diagram

Logical Operators (3)

- Conjunction
  - Symbol: ∧ (AND)
  - Meaning: logical and (both must be true)
  - Definition:
    -

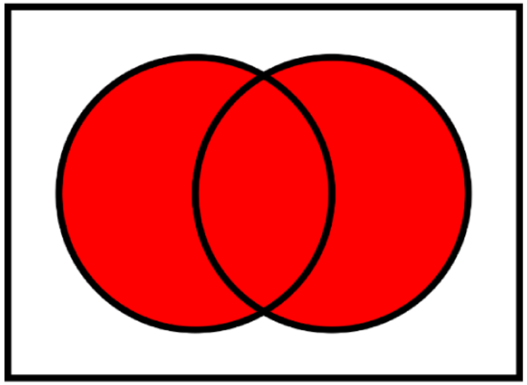
a	b	a∧b
0	0	0
0	1	0
1	0	0
1	1	1



Logical Operators (4)

- Disjunction
  - Symbol: V (OR)
  - Meaning: logical or (at least one must be true)
  - Definition:
    -

a	b	a∨b
0	0	0
0	1	1
1	0	1
1	1	1

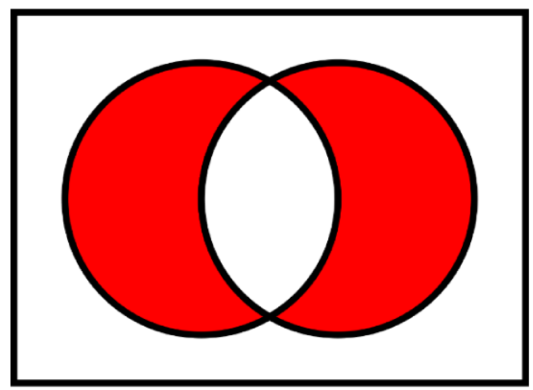


Logical Operators (5)

- Antivalence
  - Symbol: $\oplus$ (XOR, also $\nleftrightarrow$ or $\veebar$)
  - Meaning: exclusive OR (one but not both must be true)
  - Definition:

Not equivalent
(not equal)

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0



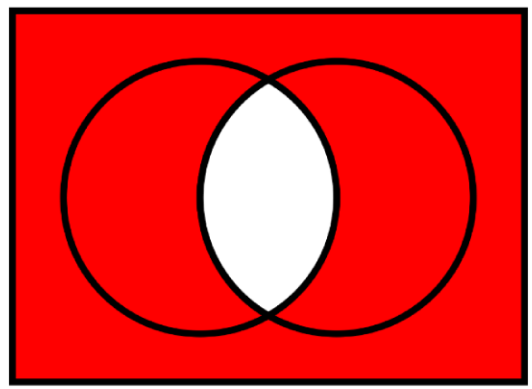
Logical Operators (6)

- Alternative Denial (Sheffer stroke)
  - Symbol: | (NAND, also $\uparrow$ or $\barwedge$)

There is no alternative

  - Meaning: negation of AND (at least one must be false)
  - Definition:
    -

a	b	a|b
0	0	1
0	1	1
1	0	1
1	1	0

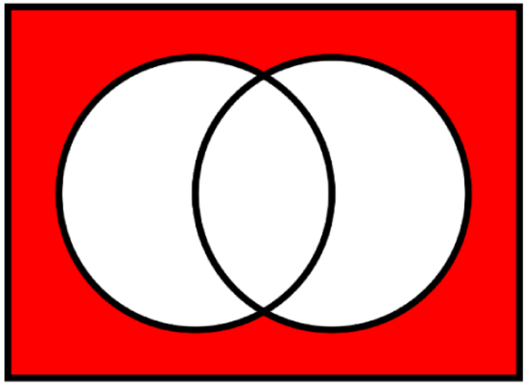


Logical Operators (7)

- Joint Denial (Peirce arrow)
  - Symbol: ↓ (NOR, also $\overline{\vee}$)
  - Meaning: negation of OR (both must be false)
  - Definition:
    -

a	b	a↓b
0	0	1
0	1	0
1	0	0
1	1	0

Nothing is true



Laws of Logic (1)

- Double Negation
  - It is not true that it is not true
    - It is true
  - Expressed as a formula
    - ¬(¬a) ↔ a
  - Proof

a	¬ a	¬ ¬ a
0	1	0
1	0	1

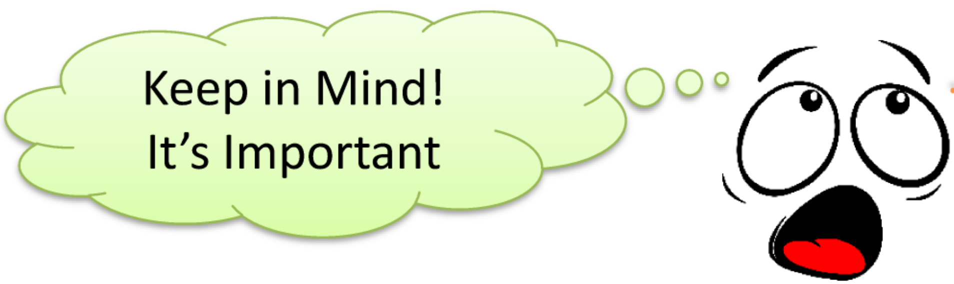
⇒ a ↔ ¬¬a

Laws of Logic (2)

- De Morgan’s Law
  - Negation of parentheses
    - Not (A and B) *is equal to* not A or not B
    - Not (A or B) *is equal to* not A and not B
  - Expressed as a formulas
    - $\neg(a \land b) \leftrightarrow \neg a \lor \neg b$
    - $\neg(a \lor b) \leftrightarrow \neg a \land \neg b$

Laws of Logic (3)

- De Morgan's Law (continued)
  - Example
    - All numbers not between 1 and 5
      - $\neg(n \geq 1 \land n \leq 5) \rightarrow \neg(n \geq 1) \lor \neg(n \leq 5) \rightarrow n<1 \lor n>5 \rightarrow \ldots,-1,0,6,7,\ldots$
      - $\neg(n \geq 1 \land n \leq 5) \nrightarrow n \leq 1 \land n \geq 5 \rightarrow \emptyset \rightarrow$ there are no such numbers



NAND Form

- Complete set of logical operators
  - NOT, AND, OR
  - NAND (or NOR)
    - Just one operator (and easy to realize on silicon)
- Transformation
  - $\neg A \rightarrow A|A$
  - $A \land B \rightarrow (A|B)|(A|B)$
  - $A \lor B \rightarrow (A|A)|(B|B)$