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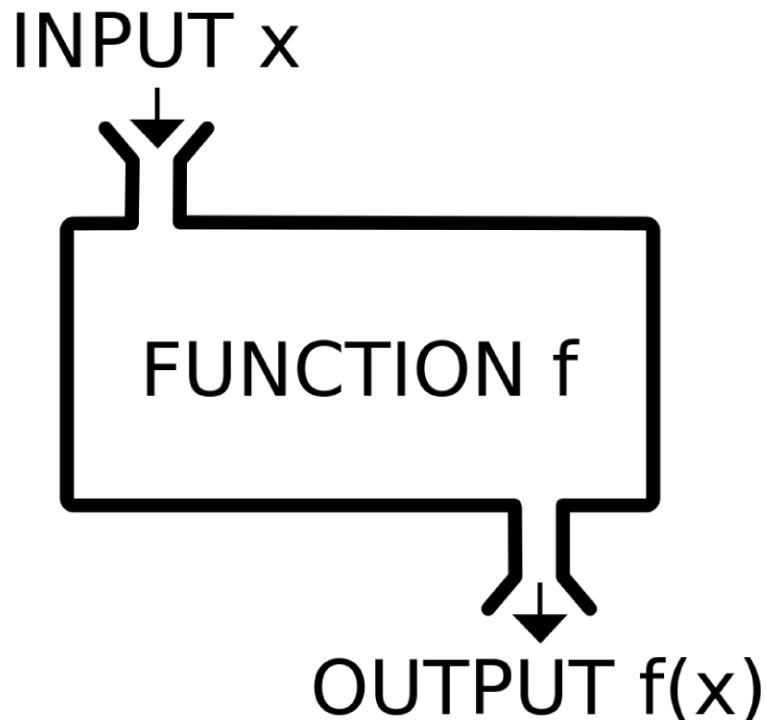
# Truth Tables

Digital Electronics

Wolfgang Neff

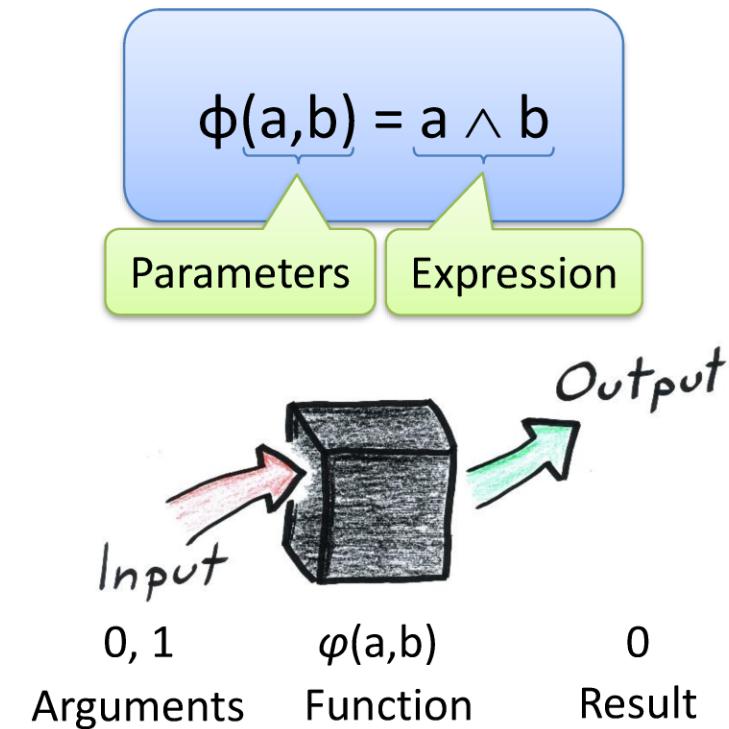
# Truth Tables (1)

- Functions
  - Process input
  - Produce output
  - They have
    - Parameters
      - Input
  - They return
    - Results
      - Output



# Truth Tables (2)

- Logical Functions
  - Are expression of
    - Parameters (a, b, c, ...)
    - Operators ( $\neg$ ,  $\wedge$ ,  $\vee$ , ...)
    - Functions ( $\varphi$ ,  $\psi$ ,  $\chi$ , ...)
  - Alternative terms
    - Logic function
    - Boolean function
    - Switching function



# Truth Tables (3)

- Logical Functions (continued)
  - Arithmetic and truth functions are different
    - Arithmetic functions are infinite
    - Truth functions are finite



$$\begin{aligned}f(1,1) &= 2 \\ f(1,2) &= 3 \\ f(1,3) &= 4 \\ f(1,4) &= 5\end{aligned}$$

...

Arguments never end

Arithmetic function  $f(x,y) = x+y$

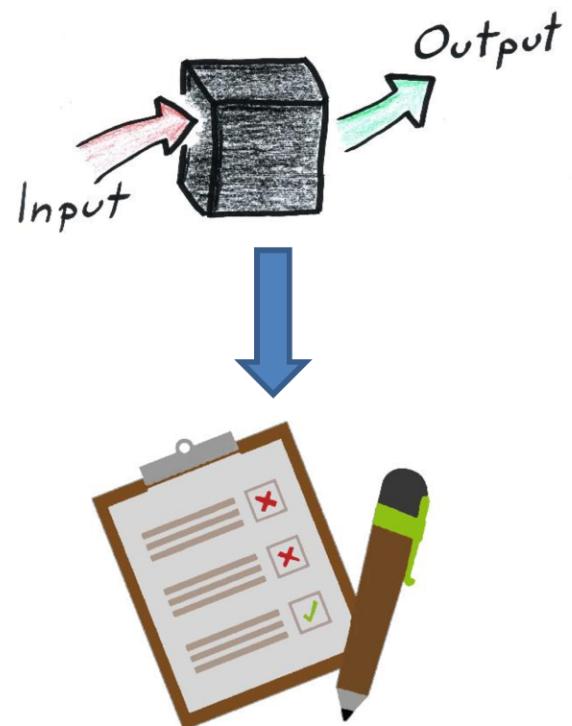
$$\begin{aligned}\varphi(0,0) &= 0 \\ \varphi(0,1) &= 0 \\ \varphi(1,0) &= 0 \\ \varphi(1,1) &= 1\end{aligned}$$

No arguments any more

Truth function  $\varphi(a,b) = a \wedge b$

# Truth Tables (4)

- Logical Functions (continued)
  - A truth table can be made
    - All possible inputs (arguments)
    - All possible outputs (results)
  - Alternative terms
    - Truth table
    - Switching table
    - State table



# Truth Tables (5)

- Logical Functions (finished)

- Example

- $\phi(a,b) = a \wedge b$



- Description

- Two parameters
      - a, b
    - Four arguments
      - (0,0), (0,1), (1,0), (1,1)
    - One result
      - 0 or 1

a	b	$\phi(a,b)$
0	0	0
0	1	0
1	0	0
1	1	1

# Truth Tables (6)

- Examples
  - Functions with one parameter:  $\phi(a)$
  - Functions with two parameters:  $\phi(a,b)$

a	$\phi(a)$
0	...
1	...

a	b	$\phi(a,b)$
0	0	...
0	1	...
1	0	...
1	1	...

# Truth Tables (7)

- Construction
  - Number of columns
    - Number of parameters:  $n$
    - Plus number of results
  - Number of rows
    - Number of arguments:  $2^n$
  - First column
    - Fifty-fifty:  $\frac{1}{2}$  column 0,  $\frac{1}{2}$  column 1

# Truth Tables (8)

- Construction (continued)
  - Second column
    - Fifty-fifty but twice as fast:  $\frac{1}{4} 0, \frac{1}{4} 1, \frac{1}{4} 0, \frac{1}{4} 1$
  - And so on ...
  - Checklist
    - First rows starts with: 0 0 0 0 ...
    - Last rows ends with: 1 1 1 1 ...
    - Last column is alternating: 0 1 0 1 0 1 ...
    - Sequence of rows: binary natural numbers (0 1 2 3 ...)

# Truth Tables (9)

- Functions with four parameters:  $\phi(a,b,c,d)$

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

a	b	c	d
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



continued on the right

# Truth Tables (10)

- Logical Equivalence
  - Truth tables are identical
  - $\phi \leftrightarrow \psi$

a	b	$\phi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

a	b	$\psi(a,b)$
0	0	0
0	1	1
1	0	1
1	1	0

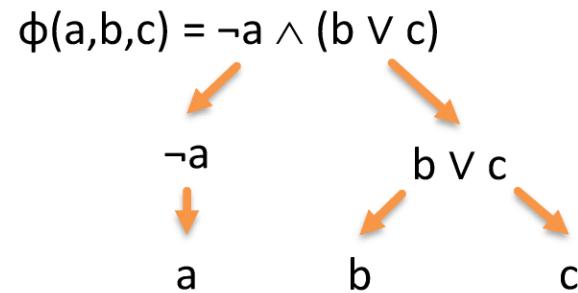
# Truth Tables (11)

- Creation (first method)
  - Mathematician's method
  - Term by term
    - Terms: compose the expression of a function
    - Functions: span a tree of terms



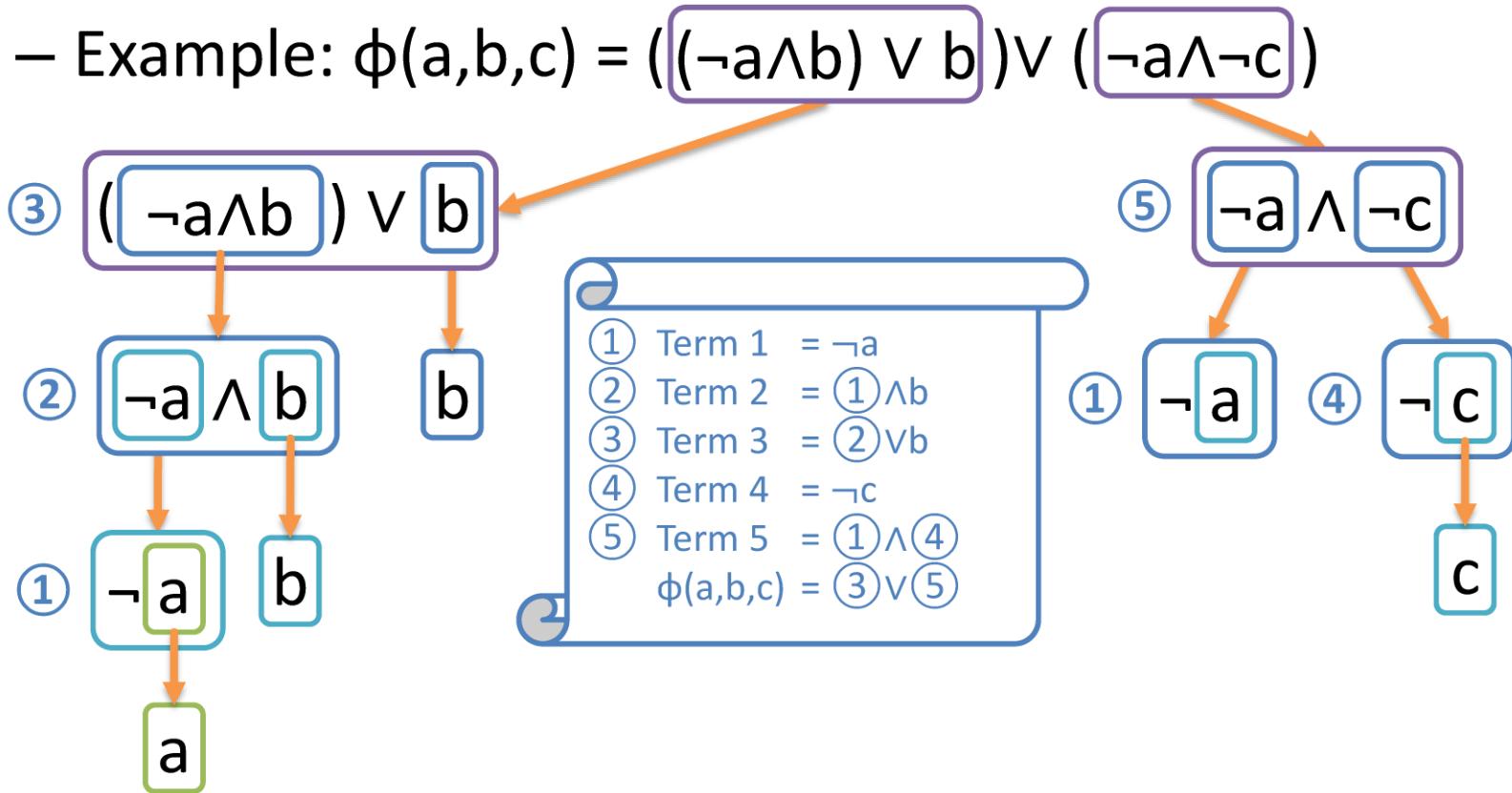
$$\phi(a,b,c) = \underbrace{\neg a}_{\text{1st term}} \wedge \underbrace{(b \vee c)}_{\text{2nd term}}$$

$\underbrace{\quad}_{\text{3rd term}}$



# Truth Tables (12)

- Creation (first method, continued)



# Truth Tables (13)

- Creation (first method, finished)

– Example:  $\phi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

a	b	c	①	②	③	④	⑤	$\phi(a,b,c)$
a	b	c	$\neg a$	$\textcircled{1} \wedge b$	$\textcircled{2} \vee b$	$\neg c$	$\textcircled{1} \wedge \textcircled{4}$	$\textcircled{3} \vee \textcircled{5}$
0	0	0	1	0	0	1	1	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	1	1	0	1
1	1	1	0	0	1	0	0	1

# Truth Tables (14)

- Creation (second method)
  - Philosopher's method
    - Put each symbol in a column
    - Fill in the arguments
    - Perform the operations
      - Fill the column with the results
      - The last column shows the result of the truth function.



a	b	$a \rightarrow$	$\wedge$	b
0	0	0	0	0
0	1	0	0	1
1	0	1	0	0
1	1	1	1	1

# Truth Tables (15)

- Creation (second method, continued)

– Example:  $\phi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

Sequence					2	1	4	3		6	5		12		8	7	11	10	9	
a	b	c	(	(	$\neg$	a	$\wedge$	b	)	$\vee$	b	)	$\vee$	(	$\neg$	a	$\wedge$	$\neg$	c	)
0	0	0			1	0	0	0		0	0		1		1	0	1	1	0	
0	0	1			1	0	0	0		0	0		0		1	0	0	0	1	
0	1	0			1	0	1	1		1	1		1		1	0	1	1	0	
0	1	1			1	0	1	1		1	1		1		1	0	0	0	1	
1	0	0			0	1	0	0		0	0		0		0	1	0	1	0	
1	0	1			0	1	0	0		0	0		0		0	1	0	0	1	
1	1	0			0	1	0	1		1	1		1		0	1	0	1	0	
1	1	1			0	1	0	1		1	1		1		0	1	0	0	1	

# Truth Tables (16)

- Don't-Care Terms
  - Truth tables can be incomplete
  - Length of truth table is fixed:  $2^n$
  - Unused rows are called don't-care terms
  - They must not be omitted
  - They are marked by X

# Truth Tables (17)

- Don't-Care Terms (continued)
  - Example: Which face of a dice has six pips?



n	a	b	c	$\phi(a,b,c)$	Face
0	0	0	0	X	invalid
1	0	0	1	0	
2	0	1	0	0	
3	0	1	1	0	
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	1	
7	1	1	1	X	invalid

# Truth Tables (18)

- Conclusion
  - Truth tables describe logic functions
    - One function has exactly one truth table
  - Truth tables are not unique
    - Many functions have the same truth table
    - These functions are logically equivalent
  - Some of these functions can easily be found
    - Truth table → logic function -> digital circuit