Combinatorial Circuits

Mechanical and Electrical Engineering

Second Grade Level

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Combinatorial Circuits (1)

- ## Design of Combinatorial Circuits
  - ### Problem: Please add two bits
    - #### Hints
      - If you add two numbers a carry can occur
      - You need not only determine the result but also the carry
      - On the other hand there can already be a carry when you add two numbers
      - You have to handle this carry, too
      - This type of circuit is called full adder
    - #### Mathematical description of a full adder
      - $\{0,1\} \times \{0,1\} \times \{0,1\} \longmapsto \{0,1\} \times \{0,1\}$
        $(a,b,c_i) \longrightarrow (c_o,r)$
        c_i: carry in; c_o: carry out; r: LSB of a+b (left most bit)

        Or for short:
        $\{0,1\}^3 \longmapsto \{0,1\}^2$

Combinatorial Circuits (2)

- ## Design of Combinatorial Circuits (continued)
  - ## – Example of a Binary Addition

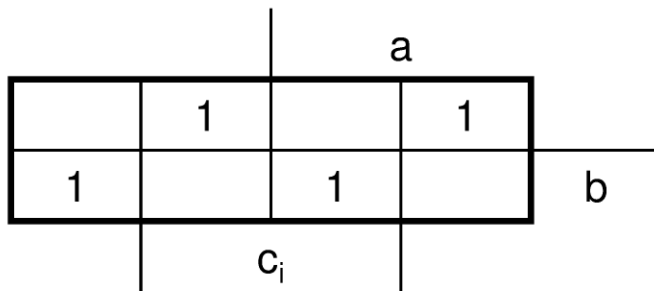| Bit position	3	2	1	0
1st Number (7)	0	1	1	1
2nd Number (5)	0	1	0	1
Carry	1	1	1	0
Result (12)	1	1	0	0

Combinatorial Circuits (3)

- ## Design of Combinatorial Circuits (continued)
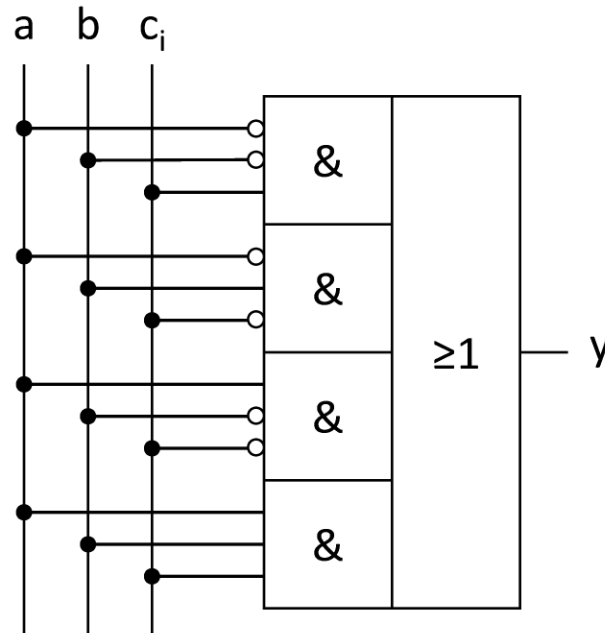 - Truth table of a one bit full adder

a	b	c_i	c_o	y
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Combinatorial Circuits (4)

- **Design of Combinatorial Circuits** (continued)
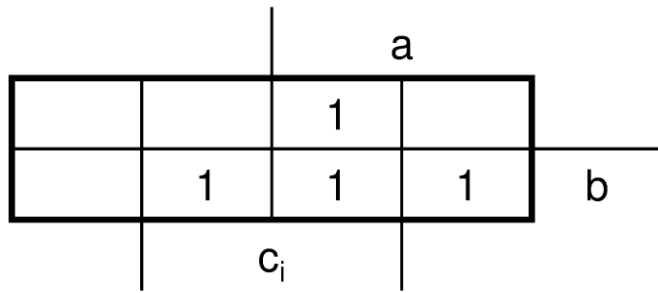 - Switching function $y(a,b,c_i)$

$t_0 = \neg a \wedge \neg b \wedge c_i$

$t_1 = \neg a \wedge b \wedge \neg c_i$

$t_2 = a \wedge \neg b \wedge \neg c_i$

$t_3 = a \wedge b \wedge c_i$

$y = t_0 \vee t_1 \vee t_2 \vee t_3$

Combinatorial Circuits (5)

- Design of Combinatorial Circuits (continued)
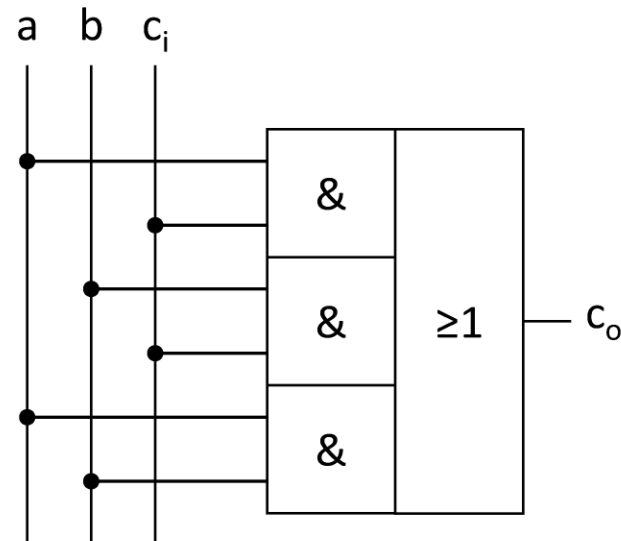 - Switching function $c_o(a,b,c_i)$

$$t_0 = a \wedge c_i$$
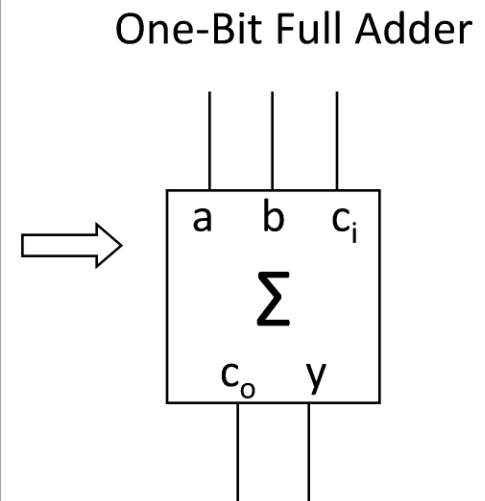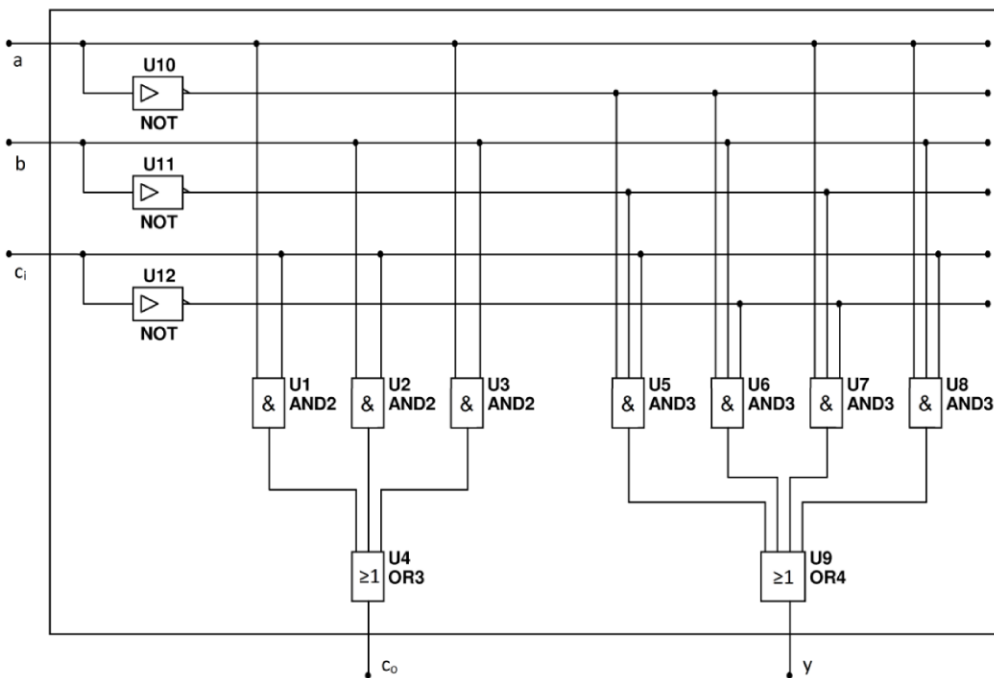$$t_1 = b \wedge c_i$$
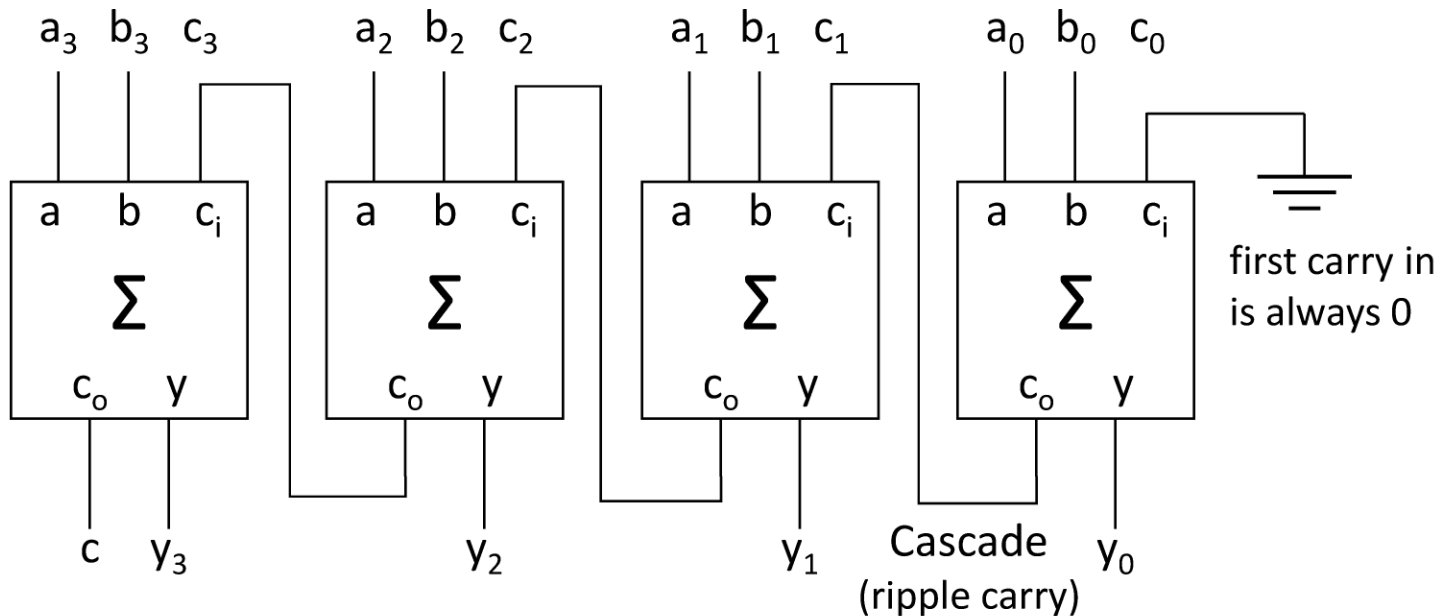$$t_2 = a \wedge b$$
$$c_o = t_0 \vee t_1 \vee t_2$$

Combinatorial Circuits (6)

- ## Design of Combinatorial Circuits (continued)
  - ### – Both functions integrated in a circuit

One-Bit Full Adder

Combinatorial Circuits (7)

- ## Design of Combinatorial Circuits (continued)
  - ### Four-bit Full Adder
    - Cascade of four one-bit full adders

Combinatorial Circuits (8)

- **Design of Combinatorial Circuits** (finished)
  - Integrated four bit half adder