Logical Operators

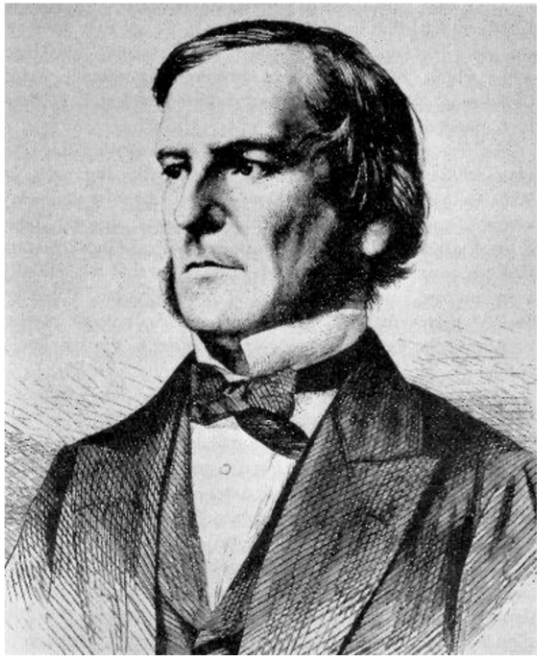
Networks and Embedded Software

Module 3.2.1

by Wolfgang Neff

George Boole

- British mathematician and philosopher
- Mathematical foundation of computer science



  - * 2 Nov 1815 in England
  - † 8 Dec 1864 in Ireland

Boolean Algebra

- A special kind of mathematics

	Arithmetic	Boolean Algebra
Values	0, 1, 2, …	0, 1 (0 → False, Low; 1 → True, High)
Operators	+, -, ·, ÷, …	¬, Λ, V, … (Negation, Conjunction, Disjunction)
Variables	x, y, z, …	a, b, c, …
Functions	f, g, … ($f(x) = x^2$)	φ, ψ, χ, … (phi, psi, chi)
Parentheses	(,)	(,)

- Example
  - $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

Logical Operators (1)

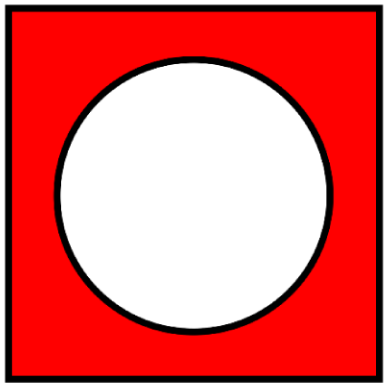
- Operate on logical values
  - True/False, On/Off, High/Low, 1/0
- Alternative nomenclature
  - Logical operator
    - Logical connective
    - Boolean operator
  - Logical value
    - Truth value
    - Boolean value

Logical Operators (2)

- Negation
  - Symbol: ¬ (NOT)
  - Meaning: logical not (contrary of …)
  - Definition:
    - $\neg : \{0,1\} \rightarrow \{0,1\}$

$$a\ \alpha \begin{cases} 1 \text{ if } a = 0 \\ 0 \text{ else} \end{cases}$$

a	¬a
0	1
1	0

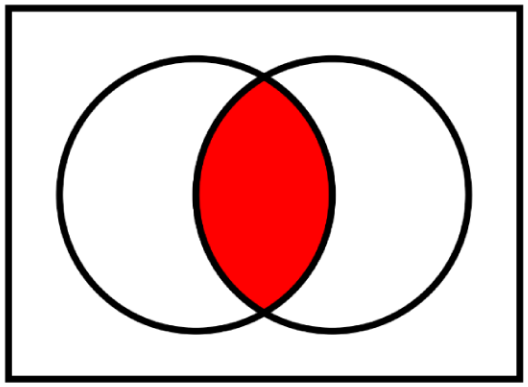


Logical Operators (3)

- Conjunction
  - Symbol: ∧ (AND)
  - Meaning: logical and (both must be true)
  - Definition:
    - $\wedge : \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

$$(a,b)\ \alpha \begin{cases} 1 \text{ if } a = b = 1 \\ 0 \text{ else} \end{cases}$$

a	b	a∧b
0	0	0
0	1	0
1	0	0
1	1	1

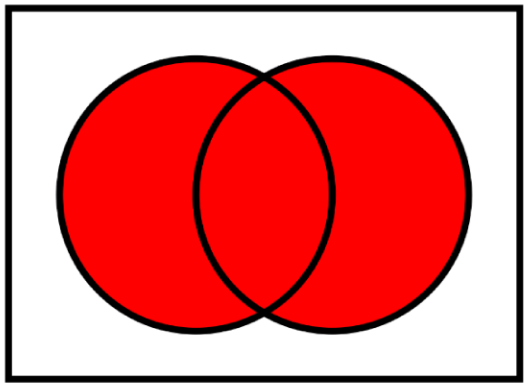


Logical Operators (4)

- Disjunction
  - Symbol: V (OR)
  - Meaning: logical or (one or both must be true)
  - Definition:
    - $\vee : \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

$$(a,b)\ \alpha \begin{cases} 0 \text{ if } a = b = 0 \\ 1 \text{ else} \end{cases}$$

a	b	a∨b
0	0	0
0	1	1
1	0	1
1	1	1

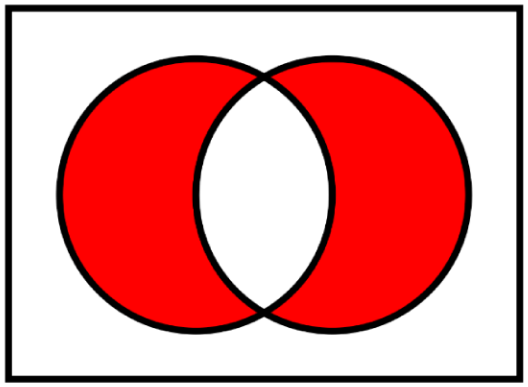


Logical Operators (5)

- Antivalence
  - Symbol: $\oplus$ (XOR, also written as $\nleftrightarrow$ or $\veebar$)
  - Meaning: exclusive OR (one but not both must be true)
  - Definition:
    - $\oplus : \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

$$(a,b)\ \alpha \begin{cases} 1 \text{ if } a \neq b \\ 0 \text{ else} \end{cases}$$

a	b	a⊕b
0	0	0
0	1	1
1	0	1
1	1	0

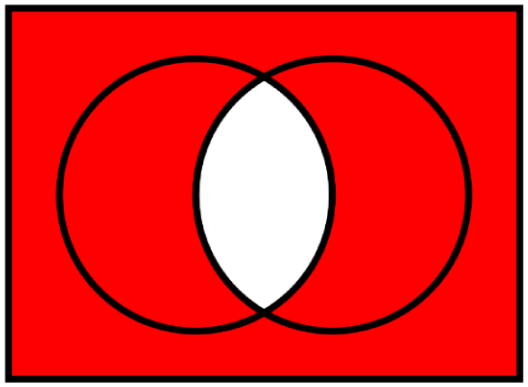


Logical Operators (6)

- Alternative Denial (Sheffer stroke)
  - Symbol: | (NAND, also written as ↑ or $\overline{\wedge}$)
  - Meaning: negation of AND (at least one must be false)
  - Definition:
    - $|: \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

$$(a,b)\ \alpha \begin{cases} 0 \text{ if } a = b = 1 \\ 1 \text{ else} \end{cases}$$

a	b	a|b
0	0	1
0	1	1
1	0	1
1	1	0

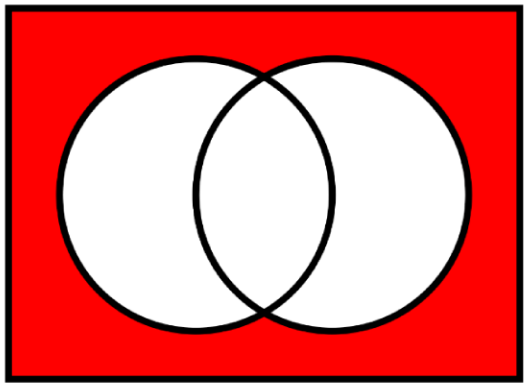


Logical Operators (7)

- Joint Denial (Peirce arrow)
  - Symbol: ↓ (NOR, also written as $\bar{\vee}$)
  - Meaning: negation of OR (both must be false)
  - Definition:
    - $\downarrow : \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

$$(a,b)\ \alpha \begin{cases} 1 \text{ if } a = b = 0 \\ 0 \text{ else} \end{cases}$$

a	b	a↓b
0	0	1
0	1	0
1	0	0
1	1	0



De Morgan's Law

- Negation of parentheses
  - $\neg(a \wedge b) \leftrightarrow \neg a \vee \neg b$
  - $\neg(a \vee b) \leftrightarrow \neg a \wedge \neg b$
- Example
  - All numbers not between 1 and 5
    - $\neg(n \geq 1 \wedge n \leq 5) \rightarrow \neg(n \geq 1) \vee \neg(n \leq 5) \rightarrow n < 1 \vee n > 5$
    - $\neg(n \geq 1 \wedge n \leq 5) \not\rightarrow n \leq 1 \wedge n \geq 5 \rightarrow \emptyset$

Keep in Mind!
It's Important!

NAND Form

- Complete set of logical operators
  - NOT, AND, OR
  - NAND (or NOR)
    - Just one operator (and easy to realize on silicon)
- Transformation
  - $\neg A \rightarrow A|A$
  - $A \wedge B \rightarrow (A|B)|(A|B)$
  - $A \vee B \rightarrow (A|A)|(B|B)$