Truth Tables

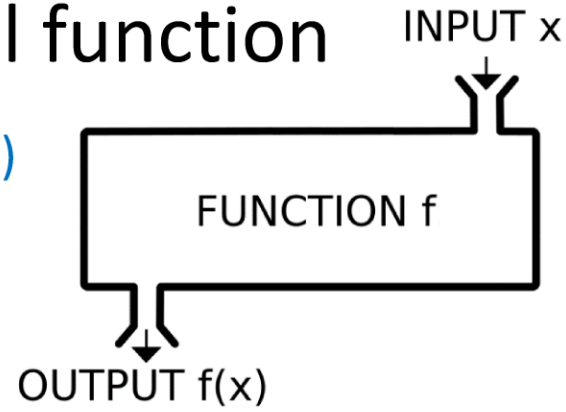
Networks and Embedded Software

Module 3.2.2

by Wolfgang Neff

Truth Tables (1)

- Representation of a logical function
  - All possible inputs (arguments)
  - All possible outputs (results)



- Example
  - $\varphi(a,b) = a \wedge b$
    - Two parameters
      - a, b
    - Four arguments
      - (0,0), (0,1), (1,0), (1,1)

a	b	φ(a,b)
0	0	0
0	1	0
1	0	0
1	1	1

Truth Tables (2)

- Functions with one parameter: φ(a)

a
0
1

- Functions with two parameters: φ(a,b)

a	b
0	0
0	1
1	0
1	1

Truth Tables (3)

- Construction guidance
  - Number of columns: number of parameters *n*
  - Number of rows: 2^n
  - Start value: *0*
  - First column: fifty-fifty (½ column 0 then ½ column 1)
  - Second column: twice as fast (¼ *0*, ¼ *1*, ¼ *0*, ¼ *1)*
  - And so on …
  - Last column: *0* and *1* alternate (010101…)

Truth Tables (4)

- Functions with four parameters: φ(a,b,c,d)

a	b	c	d
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

continues on the next table

a	b	c	d
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Logical Equivalence

- Truth functions are logically equivalent if their truth tables are identical.
- $\varphi \leftrightarrow \psi$ (φ and ψ are logically equivalent)

a	b	φ(a,b)
0	0	0
0	1	1
1	0	1
1	1	0

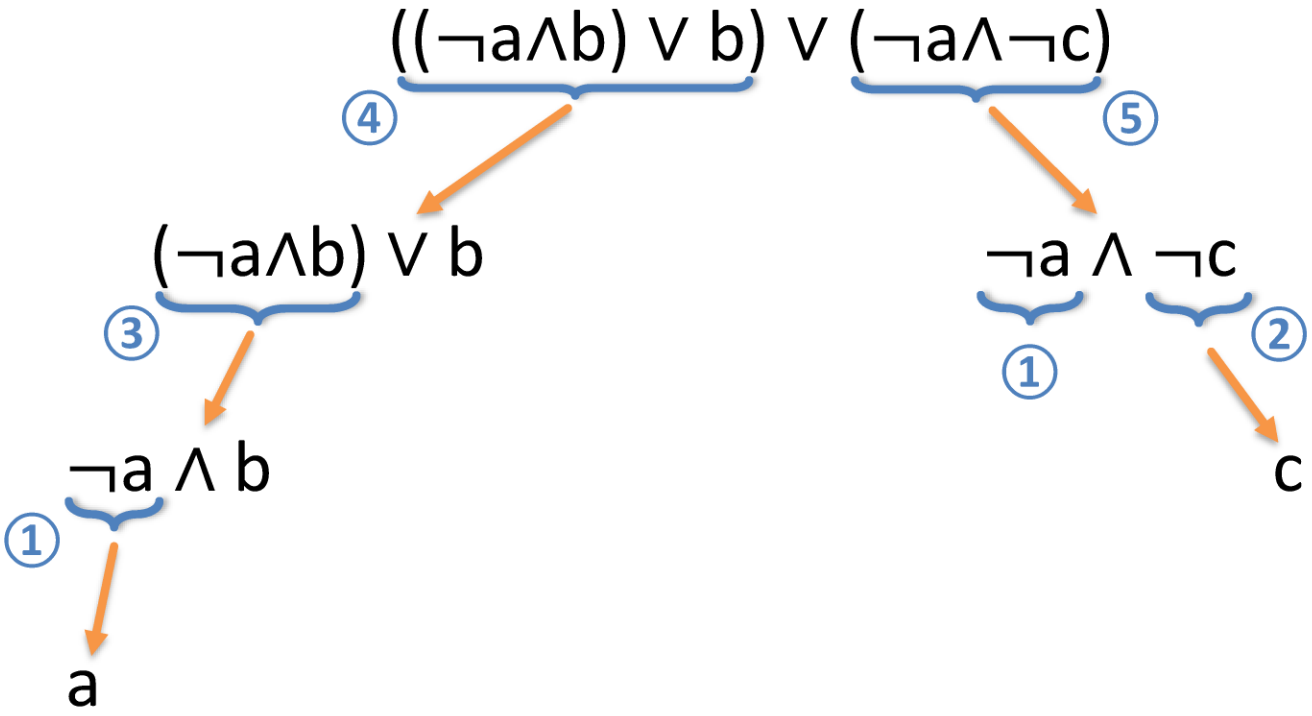
a	b	ψ(a,b)
0	0	0
0	1	1
1	0	1
1	1	0

Truth Functions (1)

- Also called
  - Logic functions
  - Boolean functions
- Domain
  - $\varphi : \{0,1\}^n \rightarrow \{0,1\}$
- Calculation
  - Term by term
    - Term: component of a function or expression

Truth Functions (2)

- Functions form a tree of terms



Truth Functions (3)

- Example: $\varphi(a,b,c) = ((\neg a \wedge b) \vee b) \vee (\neg a \wedge \neg c)$

			①	②	③	④	⑤	φ(a,b,c)
a	b	c	¬a	¬c	①∧b	③∨b	①∧②	④∨⑤
0	0	0	1	1	0	0	1	1
0	0	1	1	0	0	0	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	0	1	1	0	1
1	0	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	1	0	1
1	1	1	0	0	0	1	0	1

Truth Functions (4)

- Conclusion
  - One function has exactly one truth table
  - Are completely characterized by their truth table
  - Many functions can have the same truth table
    - These functions are logically equivalent
  - One of these truth functions can easily be found
    - Disjunctive Normal Form (DNF)

Don’t-Care Terms

- Truth tables can be incomplete.
  - Example
    - We want to use the digits 0 … 9
    - We need four variables: 2^4=16
    - Six rows are unused: 10 … 15
- Unused rows are called *don’t-care terms*
- They are marked by an X
- Important for minimization (yet to come)