

# Numeral Systems

*Please do the following exercises individually.*

## Please calculate the values of the following numbers.

$$1001_{\text{bin}} = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 9_{\text{dec}}$$

$$0110_{\text{bin}} = \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad =$$

$$1234_{\text{hex}} = \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad =$$

$$2CD9_{\text{hex}} = \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad =$$

$$6701_{\text{oct}} = \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \quad =$$

## Please calculate the representation of the following numbers.

13 ÷ 2 = 6	Rem 1	12481 ÷	=	Rem
6 ÷ 2 = 3	Rem 0	÷	=	Rem
3 ÷ 2 = 1	Rem 1	÷	=	Rem
1 ÷ 2 = 0	Rem 1	÷	=	Rem
= 1101 <sub>bin</sub>		=	_____hex	
14 ÷	Rem	1000 ÷	=	Rem
÷	Rem	÷	=	Rem
÷	Rem	÷	=	Rem
÷	Rem	÷	=	Rem
= _____bin		=	_____oct	

## Calculate the number ranges of a given number of digits.

3 hexadecimal digits:	$16^3 = 4096$	0 ... FFF <sub>hex</sub>	0 ... 4095 <sub>dec</sub>
7 binary digits:	=	0 ...	0 ... dec
5 octal digits:	=	0 ...	0 ... dec
5 binary digits:	=	0 ...	0 ... dec
8 hexadecimal digits:	=	0 ...	0 ... dec

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*Please do the following exercises individually.*

## Please calculate the values of the following numbers.

$$1001_{\text{bin}} = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 9_{\text{dec}}$$

$$0110_{\text{bin}} = 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 6_{\text{dec}}$$

$$1234_{\text{hex}} = 1 \cdot 16^3 + 2 \cdot 16^2 + 3 \cdot 16^1 + 4 \cdot 16^0 = 4660_{\text{dec}}$$

$$2CD9_{\text{hex}} = 2 \cdot 16^3 + 12 \cdot 16^2 + 13 \cdot 16^1 + 9 \cdot 16^0 = 11481_{\text{dec}}$$

$$6701_{\text{oct}} = 6 \cdot 8^3 + 7 \cdot 8^2 + 0 \cdot 8^1 + 1 \cdot 8^0 = 3521_{\text{dec}}$$

## Please calculate the representation of the following numbers.

13 ÷ 2 = 6	Rem 1	12481 ÷ 16 = 780	Rem 1
6 ÷ 2 = 3	Rem 0	780 ÷ 16 = 48	Rem 12
3 ÷ 2 = 1	Rem 1	48 ÷ 16 = 3	Rem 0
1 ÷ 2 = 0	Rem 1	3 ÷ 16 = 0	Rem 3
	= 1101 <sub>bin</sub>		= 30C1 <sub>hex</sub>

  

14 ÷ 2 = 7	Rem 0	1000 ÷ 8 = 125	Rem 0
7 ÷ 2 = 3	Rem 1	125 ÷ 8 = 15	Rem 5
3 ÷ 2 = 1	Rem 1	15 ÷ 8 = 1	Rem 7
1 ÷ 2 = 0	Rem 1	1 ÷ 8 = 0	Rem 1
	= 1110 <sub>bin</sub>		= 1750 <sub>oct</sub>

## Calculate the number ranges of a given number of digits.

3 hexadecimal digits:	$16^3 = 4096$	0 ... FFF <sub>hex</sub>	0 ... 4095 <sub>dec</sub>
7 binary digits:	$2^7 = 128$	0 ... 1111111 <sub>bin</sub>	0 ... 127 <sub>dec</sub>
5 octal digits:	$8^5 = 32768$	0 ... 77777 <sub>oct</sub>	0 ... 32767 <sub>dec</sub>
5 binary digits:	$2^5 = 32$	0 ... 11111 <sub>bin</sub>	0 ... 31 <sub>dec</sub>
8 hexadecimal digits:	$16^8 = \text{ca. 4 Mrd}$	0 ... FFFFFFFF <sub>hex</sub>	0 ... ca. 4 Mrd <sub>dec</sub>

# Binary System

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*Please do the following exercises individually.*

## What is the value of a bit at the following position<sup>1</sup>?

8th bit:  $2^7 = 128$

1st bit:

6th bit:

26th bit:

## How many bits do you need to get the following number ranges?

14 ... 19: contains 6 numbers → 3 bits needed (provides 8 numbers)

1 ... 16:

21 ... 32:

... 128:

## Please fill in the missing entries of the table.

KByte	MByte	GByte	TByte
20,971,520	20,480	20	0.020
	1,024		
			0.05
2,000,000			
		200	
	512		

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<sup>1</sup> Counting of bits starts with 1; counting of bit positions with 0. We start counting right most.

# Binary System

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*Please do the following exercises individually.*

## What is the value of the following bits<sup>1</sup>?

8th bit:  $2^7 = 128$

1st bit:  $2^0 = 1$

6th bit:  $2^5 = 32$

26th bit:  $2^{25} = 33554432$

## How many bits do you need to get the following number ranges?

14 ... 19: contains 6 numbers → 3 bits needed (provides 8 numbers)

1 ... 16: contains 16 numbers → 4 bits needed (provides 16 numbers)

21 ... 32: contains 12 numbers → 4 bits needed (provides 16 numbers)

0... 128: contains 129 numbers → 8 bits needed (provides 256 numbers)

## Please fill in the missing entries of the table.

KByte	MByte	GByte	TByte
20,971,520	20,480	20	0.020
1,048,576	1,024	1	0.00098
53,687,091	52,428	51.2	0.05
2,000,000	1953	1.90735	0.00186
209,715,200	204,800	200	0,19531
524,288	512	0.5	0,00049

<sup>1</sup> Counting of bits starts with 1; counting of bit positions with 0. We start counting right most.

# Binary Addition

*Please solve the following exercises individually.*

**Please add the following binary numbers.**

1 <sup>st</sup> Summand	1	0	1	1	0	1	1	0
2 <sup>nd</sup> Summand			1	1	0	1	0	1
Carry								
Result								

1 <sup>st</sup> Summand	1	0	1	0	1	1	1	1
2 <sup>nd</sup> Summand			1	1	1	0	0	1
Carry								
Result								

1 <sup>st</sup> Summand	1	0	1	0	1	0	1	0
2 <sup>nd</sup> Summand		1	0	1	0	1	0	1
Carry								
Result								

1 <sup>st</sup> Summand		1	1	1	1	1	1	1
2 <sup>nd</sup> Summand								1
Carry								
Result								

**Add the following binary numbers with the Windows calculator.**

$2F_{\text{hex}} + 83_{\text{hex}} =$

$3FB7_{\text{hex}} + 7007_{\text{hex}} =$

$F3_{\text{hex}} + 6A_{\text{hex}} =$

$A383_{\text{hex}} + 796A_{\text{hex}} =$

# Binary Addition

*Please solve the following exercises individually.*

**Please add the following binary numbers.**

1 <sup>st</sup> Summand	1	0	1	1	0	1	1	0
2 <sup>nd</sup> Summand	0	0	1	1	0	1	0	1
Carry	0	1	1	0	1	0	0	0
Result	1	1	1	0	1	0	1	1

1 <sup>st</sup> Summand	1	0	1	0	1	1	1	1
2 <sup>nd</sup> Summand	0	0	1	1	1	0	0	1
Carry	0	1	1	1	1	1	1	0
Result	1	1	1	0	1	0	0	0

1 <sup>st</sup> Summand	1	0	1	0	1	0	1	0
2 <sup>nd</sup> Summand	0	1	0	1	0	1	0	1
Carry	0	0	0	0	0	0	0	0
Result	1	1	1	1	1	1	1	1

1 <sup>st</sup> Summand	0	1	1	1	1	1	1	1
2 <sup>nd</sup> Summand	0	0	0	0	0	0	0	1
Carry	1	1	1	1	1	1	1	0
Result	1	0	0	0	0	0	0	0

**Add the following binary numbers with the Windows calculator.**

$$2F_{\text{hex}} + 83_{\text{hex}} = B2_{\text{hex}}$$

$$3FB7_{\text{hex}} + 7007_{\text{hex}} = AFB_{\text{hex}}$$

$$F3_{\text{hex}} + 6A_{\text{hex}} = 15D_{\text{hex}}$$

$$A383_{\text{hex}} + 796A_{\text{hex}} = 11CED_{\text{hex}}$$

# Negative Binary Numbers

*Please do the following exercises individually.*

## Please map signed numbers to their unsigned partners.

- |                    |               |                             |
|--------------------|---------------|-----------------------------|
| Range: 0 ... 15    | Negative: -2  | Positive: 14 (15=-1, 14=-2) |
| Range: 0 ... 31    | Negative: -3  | Positive: _____             |
| Range: 0 ... 127   | Negative: -5  | Positive: _____             |
| Range: 0 ... 65535 | Negative: -16 | Positive: _____             |

## Please determine the negative of the following binary numbers.

Number	0	1	0	1	0	1	0	1
Inverted	1	0	1	0	1	0	1	0
+1	0	0	0	0	0	0	0	1
Carry	0	0	0	0	0	0	0	0
Negative	1	0	1	0	1	0	1	1

Number	1	1	1	1	0	1	0	1
Inverted								
+1								
Carry								
Negative								

Number	1	0	1	0	1	1	1	1
Inverted								
+1								
Carry								
Negative								

Number		1	1	1	1	1	1	1
Inverted								
+1								
Carry								
Negative								

## Please mark the negative ones of the following numbers.

What is the rule to figure it out in your head?

- 2F<sub>hex</sub>      83<sub>hex</sub>      F3<sub>hex</sub>      6A<sub>hex</sub>      B7<sub>hex</sub>      07<sub>hex</sub>

Rule: \_\_\_\_\_

## Please determine the negative of the following binary numbers.

You may use the Windows Calculator.

- |                     |                       |
|---------------------|-----------------------|
| 2F <sub>hex</sub> : | 002F <sub>hex</sub> : |
| 6A <sub>hex</sub> : | 006A <sub>hex</sub> : |

# Negative Binary Numbers

Please do the following exercises individually.

## Please map signed numbers to their unsigned partners.

Range: 0 ... 15	Negative: -2	Positive: 14 (15=-1, 14=-2)
Range: 0 ... 31	Negative: -3	Positive: 29 (31=-1, 30=-2)
Range: 0 ... 127	Negative: -5	Positive: 123
Range: 0 ... 65535	Negative: -16	Positive: 65520

## Please determine the negative of the following binary numbers.

Number	0	1	0	1	0	1	0	1
Inverted	1	0	1	0	1	0	1	0
+1	0	0	0	0	0	0	0	1
Carry	0	0	0	0	0	0	0	0
Negative	1	0	1	0	1	0	1	1

Number	1	1	1	1	0	1	0	1
Inverted	0	0	0	0	1	0	1	0
+1	0	0	0	0	0	0	0	1
Carry	0	0	0	0	0	0	0	0
Negative	0	0	0	0	1	0	1	1

Number	1	0	1	0	1	1	1	1
Inverted	0	1	0	1	0	0	0	0
+1	0	0	0	0	0	0	0	1
Carry	0	0	0	0	0	0	0	0
Negative	0	1	0	1	0	0	0	1

Number	0	1	1	1	1	1	1	1
Inverted	1	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	0	1
Carry	0	0	0	0	0	0	0	0
Negative	1	0	0	0	0	0	0	1

## Please mark the negative ones of the following numbers.

What is the rule to figure it out in your head?

2F<sub>hex</sub>      83<sub>hex</sub>      F3<sub>hex</sub>      6A<sub>hex</sub>      B7<sub>hex</sub>      07<sub>hex</sub>

Rule: The MSB must be set. The MSB is set in 8, 9, A, B, C, D, E and F. These are all numbers above 7.

## Please determine the negative of the following binary numbers.

You may use the Windows Calculator.

2F <sub>hex</sub> :	11010001 <sub>bin</sub> = D1 <sub>hex</sub>	002F <sub>hex</sub> :	111111111010001 <sub>bin</sub> = FFD1 <sub>hex</sub>
6A <sub>hex</sub> :	10010110 <sub>bin</sub> = 96 <sub>hex</sub>	006A <sub>hex</sub> :	1111111110010110 <sub>bin</sub> = FF96 <sub>hex</sub>